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Evaluation of an empirical traction equation for forestry tires

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Abstract

Variable load test data were used to evaluate the applicability of an existing forestry tire traction model for a new forestry tire and a worn tire of the same size with and without tire chains in a range of soil conditions. The clay and sandy soils ranged in moisture content from 17 to 28%. Soil bulk density varied between 1.1 and 1.4 $\mathrm{g\,cm^{-3}}$ with cone index values between 297 and 1418 kPa for a depth of 140 mm. Two of the clay soils had surface cover or vegetation, the other clay soil and the sandy soil had no surface cover. Tractive performance data were collected in soil bins using a single tire test vehicle with the tire running at 20% slip. A non-linear curve fitting technique was used to optimize the model by fitting it to collected input torque data by modifying the coefficients of the traction model equations. Generally, this procedure resulted in improved prediction of input torque, gross traction ratio and net traction ratio. The predicted tractive performance using the optimized coefficients showed that the model worked reasonably well on bare, uniform soils with the new tire. The model was flexible and could be modified to predict tractive performance of the worn tire with and without chains on the bare homogeneous soils. The model was not adequate for predicting tractive performance on less uniform soils with a surface cover for any of the tire treatments. Published by Elsevier Science Ltd on behalf of ISTVS. All rights reserved.

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1. Introduction

New traction data were collected in at the USDA–ARS National Soil Dynamics Laboratory (NSDL) in Auburn, Alabama. Modeling tractive performance of forestry tires has been done for a limited range of tire sizes and on only a few soil

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conditions. The existing models have been utilized in computer programs for predicting skidder productivity. In the forest the soil conditions vary widely and tires are only new for a short while. The new forestry tire traction data collected in this study includes a new tire to compare with previously collected information; a worn tire, because tires are not always new; and a worn tire with chains, to include the tractive performance associated with using an aggressive forestry tire chain. Previous research using forestry tire chains has been limited to productivity studies conducted in field conditions.

Ashmore's [1] forestry tire traction model was evaluated in terms of how well predicted values of tractive performance fit the new data collected for the three tire treatments on four soils [2].

2. Literature review

Several papers have reviewed the historical development of empirical traction prediction equations [3–6]. The following dimensionless ratios are used in describing tractive performance [7]:

Gross traction ratio: $\mu_{gt} = T/rW$
Net traction ratio: $\mu_{nt} = P/W$
Motion resistance ratio: $\rho = MR/W$
where T = input torque, r = rolling radius, W = dynamic load, P = drawbar pull, and MR = motion resistance.

The Wismer-Luth equation [6] predicted the net traction ratio as a function of cone index (CI), unloaded tire diameter (d), section width (b), slip (s), and rolling radius (r) with some restrictions on tire dimensionless ratios:

$$\mu_{nt} = 0.75(1 - e^{-0.3C_n s}) - (1.2/C_n + 0.04) \tag{1}$$

where

$$C_n = (CIbd)/W$$

This equation was developed for bias-ply agricultural tires operating on cohesive and frictional soils. The wheel numeric, C_n, provides an indication of the soil–tire interaction for a given tire and load.

Clark [8] modified the Wismer–Luth equation to model field data collected with a low power agricultural tractor. Data were collected on a cleared and disked soil and undisturbed soils with short or tall grass. The Wismer–Luth coefficients were individually optimized to fit the model to the new data. The original coefficient values were reasonably good except for the coefficient value of 0.75. Clark showed that this coefficient was related to friction between the tire and soil surface. The value of this coefficient was found to be between 0.41 and 0.53 for driven wheels.

Ashmore [1] developed an empirical traction model specifically for forestry tires for data obtained on cohesive and frictional soils. The two homogenous soils were both rototilled and leveled. The clay soil had a CI of 485 kPa and the sand had a CI of 411 kPa. Moisture content of the clay was between 17 and 22%, and the sand was between 3 and 5%. Ashmore added the dynamic load ratio (W/W_r, where W_r = rated load of the tire), to the empirical model developed by Wismer and Luth [6]. Soil–tire relationships were derived as a function of C_n, s, and W/W_r. The dynamic load ratio accounts for varying dynamic loads frequently encountered during skidding operations. Ashmore's traction prediction equations for forestry tires were:

$$\mu_{gt} = 0.47(1 - e^{-0.20C_n s}) + 0.28(W/W_r) \tag{2}$$

$$\mu_{nt} = 0.47(1 - e^{-0.20C_n s}) + 0.38(W/W_r) - (0.22/C_n + 0.20) \tag{3}$$

$$\rho = -0.10(W/W_r) + 0.22/C_n + 0.20 \tag{4}$$

3. Equipment and facilities

The tire treatments used in this research were a new tire, a worn tire, and a worn tire with ring tire chains (Fig. 1). Two Firestone[1] 23. 1-26 Forestry Special (10 ply, LS-2) tires, one new and one 80% worn, were used in the soil bin tests. The ring chains leased from BABAC, Inc. added at least 7 cm to the effective lug height in the center of the worn tire.

The NSDL single-tire test vehicle [9] was used to conduct the experiment. This vehicle is instrumented to control and measure applied dynamic load, vehicle speed, tire rotational speed, travel reduction and inflation pressure. Data were collected on four soil conditions: a bare clay soil, a bare sandy soil, a clay with a pine straw cover 7–10 cm deep, and a clay sod. The four soils were selected to provide a range of soil and surface conditions (Table 1).

The Decatur clay loam, located in an enclosed building, was chosen as the bare surface clay soil. It was prepared to approximate a low-strength, undisturbed forest soil. Unlike an undisturbed forest soil, there was no ground cover or root mat that might modify the surface friction or shear strength of the soil surface. The Norfolk sandy loam was located in the same enclosed building. The Norfolk soil was prepared to approximate a firm soil surface providing favorable traction conditions. Like the Decatur soil, there was no ground cover or root mat. The two remaining clay soils were located in outside bins. The Oktibeeha clay was covered with pine straw. The Sharkey silty clay had been planted with a crop of winter rye grass about one year before the test occurred. The grass residue and new growth formed a layer of organic matter and root mat on the soil surface.

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[1] Mention of trade names is solely for the reader's information and does not imply endorsement of any product to the exclusion of others which may be suitable.

4. Data collection

Variable load tests were first run at zero net traction in each bin to determine the rolling radius (Table 2) for that soil–tire combination. The estimated rolling radii were used in all subsequent slip calculations for each tire and soil. The rolling radius estimates were made on the actual soil conditions since the tire treatments included chains to avoid problems of negative slip.

Fig. 1. Worn Firestone 23. 1-26 Forestry Special tire with ring-type forestry chains.

Table 1
Selected soil properties

	Decatur clay loam	Norfolk sandy loam	Oktibbeeha clay	Sharkey silty clay
Location composition[a]	Inside	Inside	Outside	Outside
Sand	26.9%	71.6%	20.6%	1.6%
Silt	43.4%	17.4%	18.3%	41.2%
Clay	29.7%	11.0%	61.1%	57.2%
Gravel	0%	0%	0.3%	0%
Surface cover	None	None	Pine straw	Sod
Cone index (kPa) at 140 mm	297	487	840	1418
Moisture content[b,c]	17.5%	17.6%	25.2%	28.3%
Bulk density[c] ($g\,cm^{-3}$)	1.08	1.42	1.23	1.13

[a] Batchelor [10].
[b] Dry basis.
[c] 12 Sample locations per soil at 15 cm.

Table 2
Rolling radii (m) for each soil/tire combination

Tire	Soil bin			
	Decatur	Oktibeeha	Sharkey	Norfolk
New	0.847	0.822	0.826	0.833
Worn	0.844	0.800	0.813	0.826
Worn w/chains	0.862	0.820	0.833	0.833
New w/chains	0.877	—[a]	—	—

[a] Data were not collected for this soil/tire combination.

The variable load test data were collected at 20% travel reduction. An observation included the following information: travel reduction level, dynamic load, net traction, measured travel reduction, inflation pressure, test vehicle velocity, wheel input torque, and angular velocity. These tires were operated at an inflation pressure of 138 kPa for dynamic loads varying between 0 and 32 kN at an average forward speed of 0.15 m s^{-1}.

5. Procedure

The procedure for fitting the model to the new data was based on developing a one-to-one relationship between the observed and predicted input torque using a non-linear least squares regression technique [11]. The iterative procedure optimized the model coefficients on the basis of an experimental data set by minimizing the sum of the squared errors for the entire model.

Predicted input torque (T) was expressed as a function of the net traction ratio and the motion resistance ratio: $T = (\mu_{nt} + \rho)rW$. Traction prediction equation coefficients were represented by A_i. These coefficients were treated as variables that could be optimized to improve prediction of input torque. Thus, Ashmore's equations can be written as follows.

$$\text{Net traction ratio}, \mu_{nt} = A_1\left(1 - e^{-A_2 C_n s}\right) + A_3(W/W_r) - (A_4/C_n + A_5) \tag{5}$$

$$\text{Towed force or motion resistance ratio}, \rho = (A_3 - A_6)(W/W_r) + A_4/C_n + A_5 \tag{6}$$

where the original Ashmore coefficient values were:

$A_1 = 0.47 \quad A_2 = 0.20 \quad A_3 = 0.38$
$A_4 = 0.22 \quad A_5 = 0.20 \quad A_6 = 0.48$

Each coefficient relates to a particular effect occurring in the traction process.

$A_1 =$ limit on the maximum torque that can be achieved due to friction between the tire and soil surface;

$A_2 =$ combined effect of "gripping or surface factor" (slip) and soil shear strength;
$A_3 =$ related to the magnitude of the dynamic load ratio;
$A_4 =$ related to soil strength;
$A_5 =$ models motion resistance on a hard surface at zero travel reduction;
$A_6 =$ motion resistance due to dynamic load (influenced by A_3).

Observed values for the ratios μ_{gt}, μ_{nt}, and ρ are typically greater than zero but less than one. Steel and Torrie [12] warn that observed values of ratios that vary between zero and one may have experimental errors that are not normally distributed. Thus, the assumptions inherent in least squares fitting of an equation may be violated if any of these ratios were used as the dependent variable. So pull (P), motion resistance (MR), or input torque (T), may be more appropriate dependent variables for least squares fitting of traction equations. Input torque (T) and pull (P) were both measured and input torque (T) was chosen as the dependent variable.

Since $T = (\mu_{nt} + \rho)rW$, then substitution of Ashmore's equations gives

$$T = \left[A_1\left(1 - e^{-A_2 C_n s}\right) + (2A_3 - A_6)(W/W_r)\right]rW \tag{7}$$

This shows that the coefficients A_4 and A_5 are not in this input torque (T) equation; so A_4 and A_5 will be assumed to remain unchanged from the original Ashmore's equations (i.e. $A_4 = 0.22$, $A_5 = 0.20$). Moreover, it is not possible to obtain unique values for A_3 and A_6. However, if $(2A_3 - A_6)$ is replaced by a new coefficient, α, then α can be uniquely determined. If we assume that A_3 also remains unchanged from the original Ashmore's equation (i.e. $A_3 = 0.38$), then we can get a unique value for A_6. With these assumptions, Eq. (7) becomes:

$$T = \left[A_1\left(1 - e^{-A_2 C_n s}\right) + \alpha(W/W_r)\right]rW \tag{8}$$

Also, since the data were collected with little variation in slip (approximately 20% slip) or cone index (CI) within a given soil, very little change in A_1 and A_2 would be expected. Ashmore's values of the coefficients A_1 through A_6 were used as starting estimates of the coefficients in the optimization procedure.

Restrictions on input data made to eliminate meaningless data were:

Observed Input Torque: $T_o > 0.0$
Slip: $s > 0.0$
Observed Tractive Efficiency: $0.0 < TE_o \leq 1.0$

6. Results and discussion

6.1. New tire

The results of the least squares fitting on the new tire input torque data starting with Ashmore's original coefficients are summarized in Table 3. The optimized coefficient values differed by less than 10% from Ashmore's original values for the

Table 3
Summary of original Ashmore coefficients and optimized coefficients for input torque data for the new 23.1-26L forestry tire

Equation coefficient		Optimized				
	Original	Decatur	Norfolk	Oktibeeha	Sharkey	All[a]
A_1	0.47	0.52	0.49	0.58[b]	0.42	0.50
A_2	0.20	0.22	0.21	0.17[b]	0.21	15.15[b]
A_6[c]	0.48	0.36	0.50	0.60[b]	0.59	0.74[b]
R_2		0.9567	0.9679	0.9189	0.9602	0.9684
Residual mean square		0.8790	0.3967	0.7868	0.5583	0.4461
Number of observations		188	157	169	170	684

[a] New tire data on all four soils.
[b] Optimized values differ by more than 10% from the original value of the coefficient.
[c] $A_6 = 0.76 * \alpha$.

Norfolk soils and two of the three coefficients differed by less than 10% for the Decatur soil. All coefficients differed by more than 10% for the Oktibeeha and only one coefficient for the Sharkey soil was more than 10% different. When all the data were optimized together, the coefficient values lost some of their meaning and validity since some were negative or unreasonably large due to the variation between the different soils which cannot be adequately described by the CI.

Fig. 2 illustrates the improved prediction of tractive performance measures for the new tire data in the Decatur soil. Predicted and observed values versus dynamic load are shown using both the original coefficients and the optimized coefficients listed in Table 3. As shown in Fig. 2, the traction ratios are not affected much in the observed range of dynamic load. It would have been advantageous to extend the range of dynamic loading in addition to running the tests at several levels of slip. However, space and equipment limitations did not make these additional tests feasible.

6.2. *Worn tire with and without chains*

The results of the least squares fitting procedure of the worn tire with and without chains are summarized in Table 4. The optimized coefficients differed by less than 10% from their original values for both worn tire treatments run in the Norfolk soil and two of three differed by less than 10% from their original values in the Decatur soil. In the Oktibeeha soil, the optimized coefficients for the worn tire without chains were also within 10% of their original values. Coefficient A_1 for the worn tire with chains differed from the original values by more than 10% of the original value in the Oktibeeha soil. No optimized coefficients were within 10% of their original values for the worn tire with chains in the Sharkey soil. Fig. 3 shows predicted and observed values versus dynamic load of the tractive performance measures for the worn tire with chains in the Decatur soil.

The data for the worn tire with chains has a great deal of variation in net traction as well as input torque. The net traction ratio was always less than the gross traction

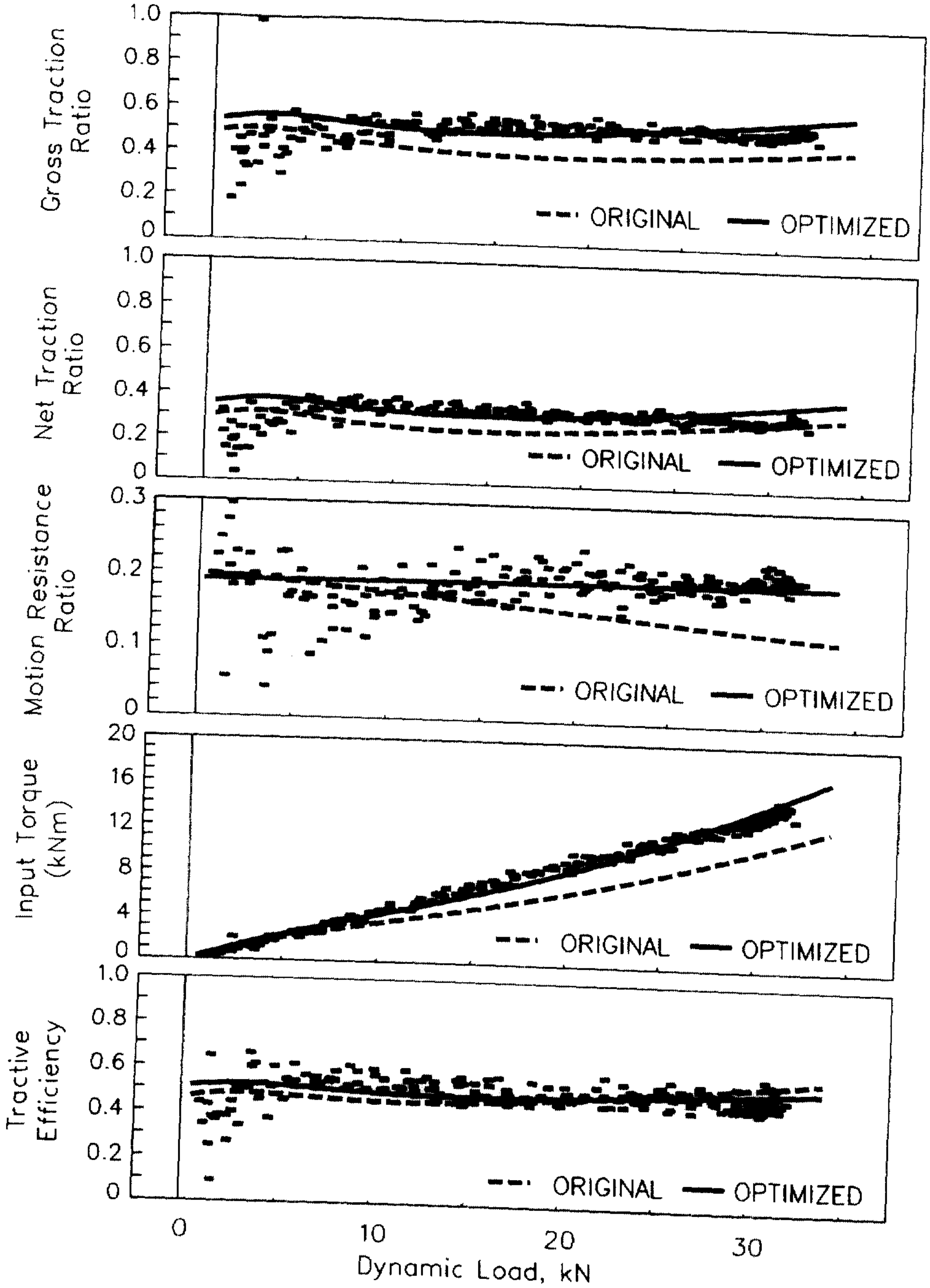

Fig. 2. Tractive performance measures for the new tire on Decatur clay loam at 20% travel reduction.

Table 4
Summary of original Ashmore coefficients and optimized coefficients for input torque data for the worn 23.1-26L forestry tires with or without tire chains

Equation coefficient		Optimized							
	Original	Decatur		Norfolk		Oktibeeha		Sharkey	
		Worn	Chains	Worn	Chains	Worn	Chains	Worn	Chains
A_1	0.47	0.52	0.52	0.45	0.48	0.45	0.54[a]	0.54[a]	0.77[a]
A_2	0.20	0.22	0.22	0.22	0.20	0.19	0.18	1.25[a]	1.94[a]
A_6[c]	0.48	0.38	0.35	0.46	0.44	0.50	0.53	0.79[a]	0.92[a]
R^2		0.9539	0.9432	0.9415	0.9480	0.9310	0.9083	0.9852	0.9535
Residual Mean Square		0.9153	1.1780	0.7813	0.7686	0.7369	1.0575	0.2444	1.0173
Number of Observations		195	184	171	152	349[b]	177	388[b]	410[b]

Optimized values differ by more than 10% from the original value of the coefficient.

[b] Two test runs were conducted for this soil–tire treatment combination.

[c] $A_6 = 0.76 * \alpha$.

ratio in the prepared soils (Decatur and Norfolk) as expected. For parts of each test in the Oktibeeha and Sharkey soils, the net traction ratio exceeded the gross traction ratio. This may be indicating that the estimate of rolling radius in these soils for the worn tire with chains was too small.

6.3. Prediction improvement

Prediction improvement for μ_{gt}, μ_{nt}, ρ, T and TE (tractive efficiency) when the optimized coefficients are used instead of the original coefficients is summarized in Table 5 for all tire treatments. Residual sums of squares (SS) were used to quantify the improvement obtained by using the optimized coefficients. The SS for the optimized coefficients were compared to the SS for the original coefficients.

In all cases, prediction of μ_{gt} was improved as might be expected since the sum of squares of differences between the observed and predicted input torque was minimized. The associated prediction of μ_{nt} also improved except for the bare worn tires in the Oktibeeha and Sharkey soils. The slight improvement in prediction of ρ and TE for the new tire data in the Decatur soil as well as the lack of improvement in all other cases has little meaning since the associated coefficients were not directly included in the optimization procedure. Generally, the most improvement was shown for all tire treatments in the Decatur soil. The least improvement occurred in the Norfolk soil. Overall, prediction of μ_{nt} in the Oktibeeha and Sharkey soils was not improved using the optimized coefficients. Because the optimized coefficients were developed using a non-linear least squares optimization on input torque (T) data, the optimized coefficients guarantee an improvement in the prediction of T but may not give an improvement in the other performance variables. The optimized coefficients for the Norfolk data improved prediction by only 7%, indicating that the original coefficients are fairly good for predicting input torque for that particular soil. This result is supported by the fact that the Norfolk soil had characteristics that

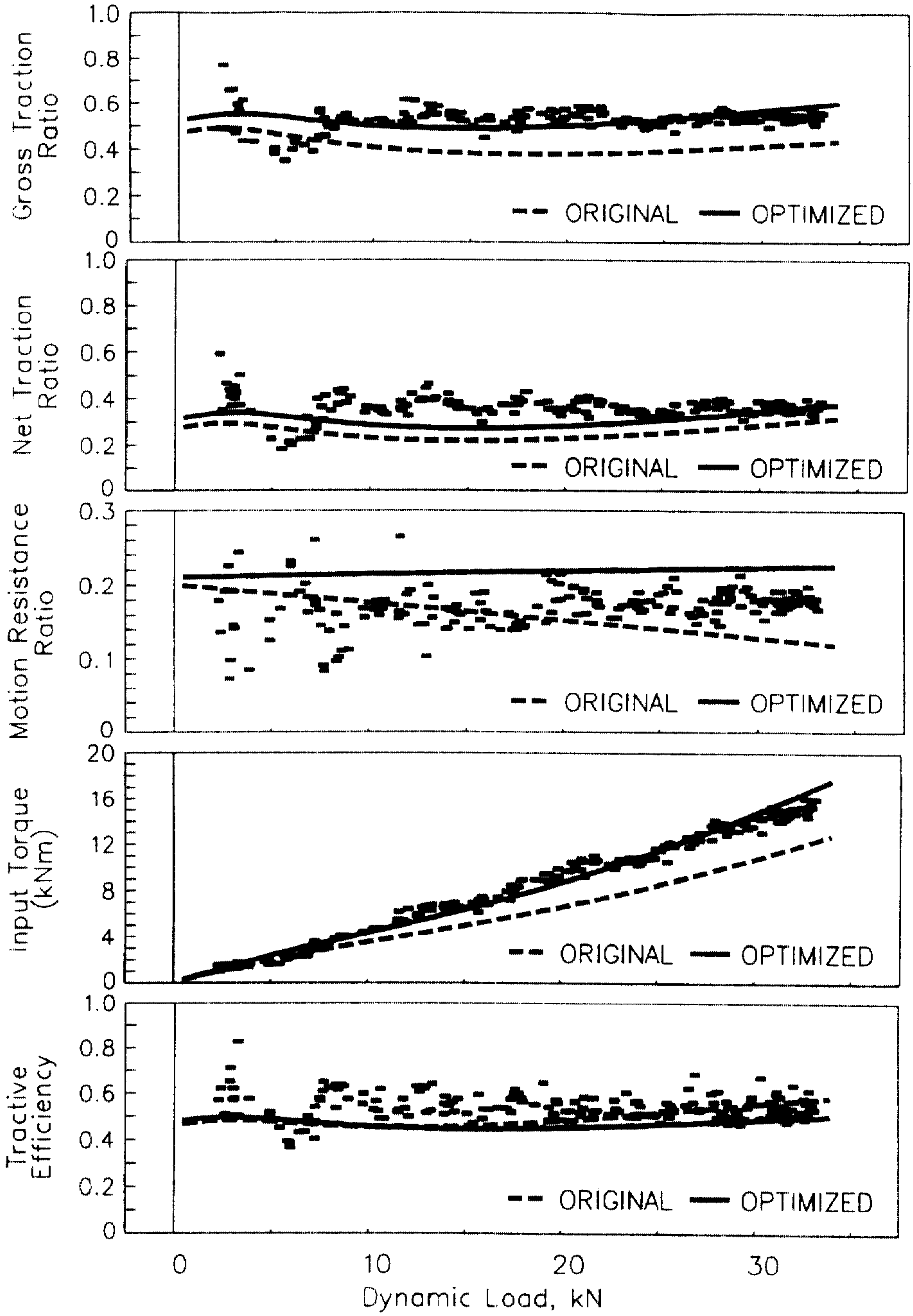

Fig. 3. Tractive performance measures for the worn tire equipped with chains on Decatur clay loam at 20% travel reduction.

Table 5
Prediction improvement using optimized coefficients for data collected for the new tire, and the worn tire with and without chains

Variable	Decatur versus original[a]			Norfolk versus original			Oktibeeha versus original			Sharkey versus original		
	New	Worn	Chains	New	Worn	Chains	New	Worn	Chains	New	Worn	Chains
%[b]												
μ_{gt}	178.69	346.98	125.02	6.16	9.91	5.73	84.22	23.73	53.84	618.76	238.51	64.85
μ_{nt}	93.15	106.83	104.82	2.16	1.16	9.72	40.56	−12.69	27.90	45.88	−46.41	−85.46
ρ	94.98	−15.84	−30.99	−20.84	−10.17	2.03	−62.80	9.60	−14.18	−11.75	−56.46	−96.66
TE	31.70	4.21	−18.93	−10.89	−3.73	−5.83	−46.01	−6.90	0.00	−34.82	−58.51	−95.28
T	571.93	486.36	562.36	7.11	7.11	53.60	106.83	97.02	26.27	1072.95	2212.94	161.38

[a] $(SS_{\text{original}} - SS_{\text{optimized}})/SS_{\text{optimized}} \times 100\%$ where $SS_i = \Sigma(\hat{y}_i - y_o)^2$, $\hat{y}_i$ = predicted value and y_o = observed value.

[b] A positive value indicates the optimized coefficients improved the predicted value; a negative value indicates that the original coefficients provided a better prediction.

were very similar to the soils Ashmore used to develop his model. The improvement shown in the other soils indicates that the original coefficients may not be adequate for a wider range of soil and surface conditions.

6.4. *Coefficient sensitivity*

The modified coefficients were subjected to an individual one-at-a-time sensitivity analysis [2]. Values of μ_{gt}, μ_{nt}, ρ, T, and TE were predicted by each set of coefficients, with one of the modified coefficients replaced by its original value. The residual SS between the predicted and observed values were computed.

The analysis showed that predicted values of μ_{gt}, μ_{nt}, and T were most influenced by coefficient A_1. The optimized value of A_1 varied between 0.42 and 0.58 for the new tire. This range of values agreed with findings by Clark [8]. The equation for gross traction ratio, μ_{gt}, did not include coefficients A_4 and A_5. Likewise, for the input torque, T, the values of A_4 and A_5 had no effect. A_6 had no effect on the net traction ratio, μ_{nt}. The motion resistance ratio, ρ, did not include A_1 and A_2.

7. Tire comparison and discussion

The most improvement from the optimization procedure was shown in the prediction of μ_{nt} and μ_{gt} for the worn tires in the Decatur soil and for the new tire in the Oktibeeha and Sharkey soils. Overall, the least improvement was associated with the Sharkey soil and the greatest in the Decatur soil for the three tire treatments. The Norfolk soil also showed improved prediction of μ_{nt}, μ_{gt}, and ρ but the magnitude of improvement was less than for the other soil types.

Evaluation of this traction model illustrates differences between "laboratory" and "field" conditions. The Decatur clay loam and the Norfolk sandy loam are typical laboratory test conditions. The Oktibeeha clay with a layer of pine straw and the Sharkey silty clay with sod cover approximated forest field conditions. The predictive ability of an existing forestry tire traction model was not flexible enough to successfully model the tractive performance in the Sharkey and Oktibeeha soils due to the combined effects of the soil condition and the soil surface. However, the model performed well for all tire treatments in the Decatur and Norfolk soils.

8. Conclusions

The new data were collected on four additional soil conditions with a different sized forestry tire in a new condition and a worn condition. The adequacy of the original Ashmore equation and their coefficient values were illustrated by how well the new tire data collected in the Decatur and Norfolk soils were modeled. The flexibility of the model was shown in how well the worn tire with and without tire chains in the same two soils was modeled. The limitations of the model become apparent when it was used to predict tractive performance on much stronger soils with surface covers of pine straw or sod.

9. Recommendations for future research

This research has quantified the tractive performance of tires operating with and without chains on various soil conditions. Evaluation of an existing forestry tire traction model shows that it may not be adequate for forestry field conditions but may be appropriate for good traction conditions, such as might be found on constructed skid roads. Future research efforts in tractive performance and modeling of forestry tires with chains should include "field" studies on forest soils as well as "laboratory" studies over a wider range of soil conditions (to vary C_n) and drawbar loads (to vary slip) than was possible in this research. Additional effort should be directed toward development of a more flexible traction model that accounts for varying soil surface conditions as encountered in the field. This type of model could aid forest and equipment managers in choosing the most appropriate machine options for a particular operation.

Forestry traction research is important in helping us better understand how mechanized forest operations impact the soil and water resources that support a forest. Expanding forestry traction research to a wider range of soil conditions and investigating site impacts will provide valuable information that will allow forest managers to be better stewards of our forest lands.

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