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Journal of the Indian Society of Soil Science, Vol. 41, No. 4, pp 730-737 (1993)

A Reassessment of the Crop Tolerance Response Function

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Abstract: ***Crop salt tolerance data have traditionally been analyzed with a three parameter threshold-slope model that assumes maximum yield until salinity threshold value, and a linear decrease in yield beyond the threshold. This study shows that an alternative, S-shaped response model provides** an **equally good or** better fit **to many experimental** data sets, **and with less bias in the parameter estimation process. Analysis of 204 data sets from the original database compiled by Mass and Hoffman (1977) revealed that a single dimensionless curve could be used to represent the salt tolerance of most crops. The curve is given by** $Y_r = 1/[1+(c/c_{50})^3]$, where Y_r is the **relative yield, c the** average root zone **salt** concentration, **and** c_{50} a **parameter which &scribes** the degree **of salt tolerance of the crop (the average root zone salinity at which the yield has declined by 50%). The presence of a unique dimensionless curve to describe the salt tolerance of many or most crops may point to some common mechanism that could govern the yield response of crops to salinity, andperhaps** to **other yield limiting** factors as **well.*** (Key words: Salt tolerance, response function, soil salinity, yield-limiting factors)

Salinity-induced crop yield losses may often be prevented by adopting soil and water management and agronomic practices that are appropriate for the local soil, crop and environmental conditions. Unfortunately, high salinities are sometimes difficult to prevent because of a lack of non-saline irrigation water. When and where salinity is a problem, an effective use of available soil and water resources requires the production of agricultural crops that are relatively tolerant to salinity. To do so, reliable data are needed to predict crop yields in response to various levels of salinity in the root zone. Numerous field and laboratory experiments have been conducted over the past 80 years or so to obtain the necessary salt tolerance data. This has resulted in various salt tolerance lists (Harris & Pittman 1919; van den Berg 1950; U.S. Salinity Laboratory Staff 1954; Bernstein-1964; de Forges 1970; Maas & Hoffman 1977; among others).

Probably the most comprehensive analysis to date of available salt tolerance data was published by Maas and Hoffman (1977). and recently updated by Maas (1991). Based on an extensive literature search, Maas and Hoffman (1977) concluded that crop yield as a function of the average root zone salinity could be described reasonably well with a piecewise linear response function characterized by a salinity "threshold" value below which the yield is unaffected by soil salinity, and above which yield decreases linearly with salinity. Results compiled by Maas and Hoffman (1977). and since then by many

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others, also show that the crop salt tolerance response function is variety-specific, and may depend, among other things, on the unique soil, environmental and water management conditions of an experiment.

The threshold-slope model of Maas and Hoffman (1977) has proved to be extremely useful for a variety of applications in research and management. Notwithstanding the popularity of this model, other salinity response functions have been found equally successful in describing observed crop salt tolerance data (Feinerman et al. 1982; van Genuchten & Hoffman 1984). The study by van Genuchten and Hoffman (1984). in particular, pointed to potential problems with the threshold-slope model in describing experimental data, namely the relatively poor definitidn of the salinity threshold value for data sets which are poorly defined, erratic or have limited observations, and the inability to accurately reproduce many salt tolerance data sets at relatively high soil salinities.

In this paper we will use one of the smooth S-shaped response functions of van Genuchten and Hoffman (1984) to re-analyze the salt tolerance database of Mass and Hoffman (1977). In particular, we will illustrate the improved accuracy of this function in describing several data sets, and also show how the function was used to derive a single dimensionless (scaled) salt tolerance response

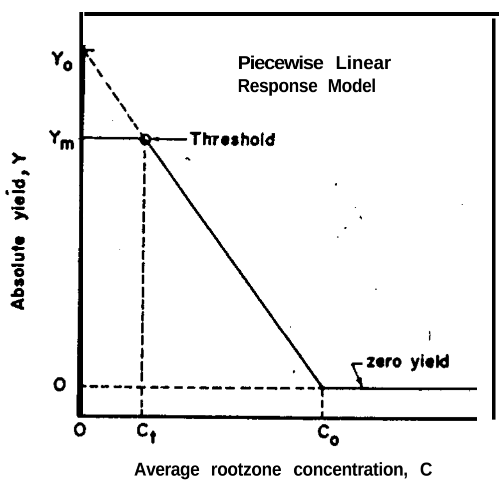

Fig.1. Graphical representation of the piece-wise linear crop and salt tolerance response function (Eq.1)

model applicable to most data sets considered in the analysis.

Theory

Threshold-slope salt tolerance response function: The threshold-slope model of Maas and Hoffman (1977) is characterized mathematically by a piecewise linear function (Fig.l) containing three independent parameters: the maximum yield under nonsaline conditions (Y_m), the salinity threshold (c_t), which is defined as the maximum soil salinity without yield reduction, and the slope(s) of the function determining the fractional yield decline per unit increase in salinity beyond the threshold. In equation form, the threshold-slope model is given by

$$Y = \begin{cases} Y_m & 0 < c < c_t \\ Y_m - Y_m\, s(c - c_t) & c_t < c < c_0 \\ 0 & c > c_0 \end{cases} \quad \ldots (1)$$

where, Y is crop yield, c is the average root zone salinity, and c_o the concentration beyond which the yield is zero. Soil salinities in Eq. (1) can be expressed in terms of concentration, osmotic potential, or electrical conductivity of either the soil water extract (EC_{sw}) or the soil saturation extract (EC_e). We note that c_o is not an independent parameter, but can be expressed in terms of c_t and s as follows:

$$c_o = c_t + 1/s \quad \ldots (2)$$

Maas and Hoffman (1977). among others, found that Eq.(l) gives a fairly good description of many salt tolerance data sets. Unfortunately, the function has at least two shortcomings, one of which is illustrated in fig. 2, using barely (forage) data obtained by Saini (1972). Notice that the yield data are expressed as a fraction of the control yield, Y_c, of the first data point at the lowest salinity. ,The solid curve in fig. 2b was obtained with a nonlinear least-squares parameter optimization programme, SALT (van Genuchten 1983). which simultaneously fits the three unknown coefficients (Y_m, c_t and s) of the threshold-slope model (Eq.1) to the observed data. The SALT program minimizes the objective function

$$0(b) = \sum_{i=1}^{i=n} [Y_i - \hat{Y}_i (b)]^2 \quad \ldots (3)$$

where, Y_i and $\hat{Y}_i$ are measured and calculated crop

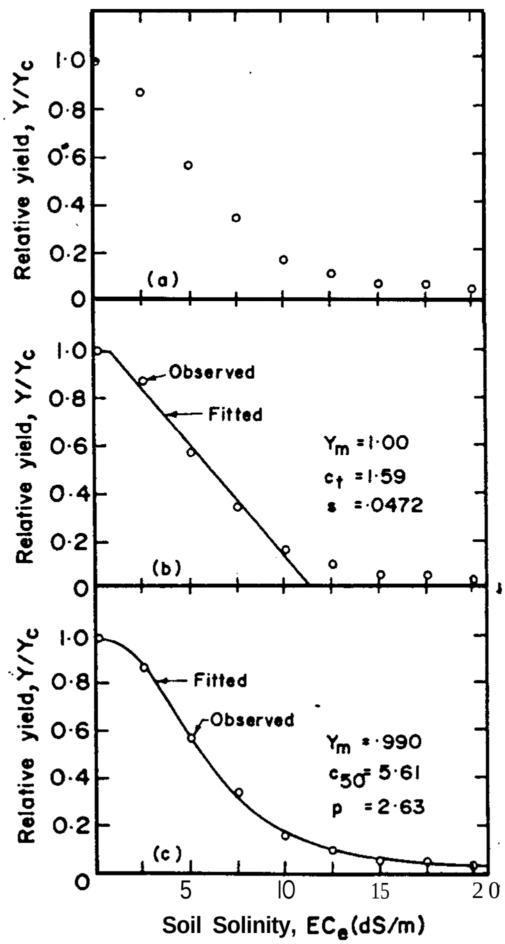

Fig.2. Observed salt **tolerance data for barley (a), and fitted response functions using the threshold slope model (b) and the S-shaped model (c)**

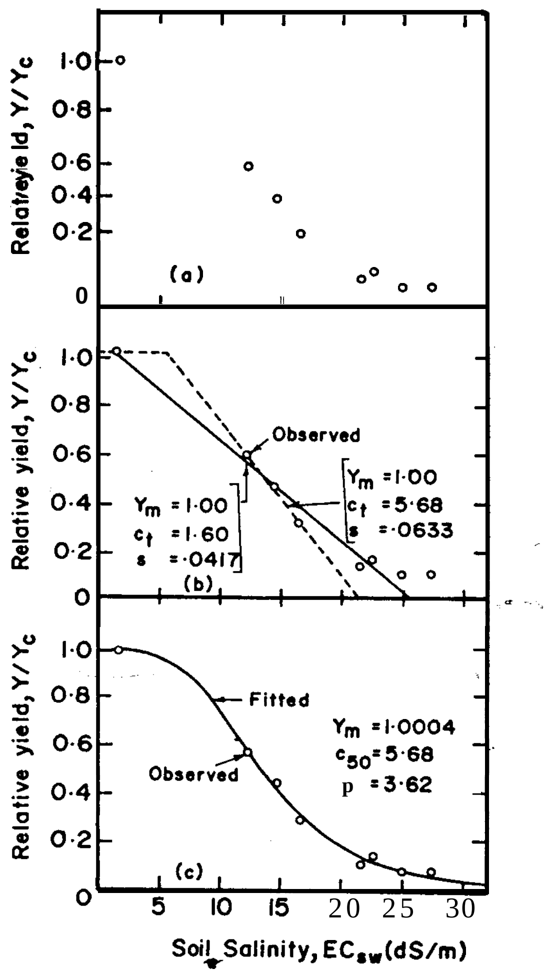

Fig.3. Observed salt tolerance response data for meadow foxtail (a) and fitted response **functions using the Mold-slope model (b) and the S-shaped model (c)**

yields at the n measured soil salinity c_i and b is the vector of unknowns coefficient: b = $\{Y_m, c_t, s\}$. Figure 2 shows that the salinity threshold-slope model matches the data reasonably well at the lower salinities. However, the data at the higher salinities, notably when c > c_o, are not described well. The tailing phenomenon at the higher soil salinities is a characteristic feature of many salt tolerance data sets and cannot be described with the piecewise linear threshold-slope model.

A second weakness of the threshold-slope model is illustrated in fig. 3 using data for Meadow Foxtail grass (Brown & Bernstein 1953). Note that the experimental curve in this case is poorly defined at the higher yields. Straightforward application of the three-parameter inversion method to these data resulted in a threshold salinity value, c_t, that was considerably less than the salinity of the first measured data point (fitted curve not shown in fig. 3b). This situation, found to be typical of many salt tolerance data sets (van Genuchten & Hoffman 1984). leads to a uniquely defined response curve only for salinities exceeding the threshold value. However, the fitted values of Y_m and c_t are meaningless in these types of examples since no data points at the lower salinities are available to fix the two parameters. In fact, different initial estimates for Y_m and c_t in the least- squares analysis will usually lead to different fitted values for these two coefficients.

There are several ways to resolve this problem, each one having its disadvantages. One possibility would be to assume that either c_t is known before hand and equal to the salinity of the first data point, or that Y_m is known and coincides with the yield of that same first point. Both assumptions will fix the the end point of the fitted curve in the upper left point of fig. 3b. Unfortunately, the approach results either in a Y_m value that is still less than the yield of the first data point or in a threshold salinity that still lies to the left of the point. Both situations appear unrealistic and are not further considered here.

A somewhat more realistic approach would be to fix both c_t and Y_m at the values of the first data point in fig.3. This alternative leads to a one parameter fit for s and results in the solid curve of fig.3b. Yet another approach is possible by reasoning that one should be interested mainly in the higher yields, and hence that the outliers at the higher salinities (the last four data points) should be neglected in the analysis. This reasoning leads to the dashed curve. Notice that the threshold value is now three to four times larger than the threshold value for the solid line. This example demonstrates that the threshold- slope model in conjunction with the poorly defined data set can quickly lead to ambiguities that are not easily resolved.

Alternative S-shaped salinity response model : In order to avoid some of uniqueness problems with the threshold-slope model, we examined several alternative response functions which would give a more accurate description of the above data, and also would lead to a more stable and unbiased inverse problem. A preliminary analysis of several candidate functions, including exponential functions of the type by Feinerman *et al.* (1982), caused us to select the following smooth, sigmoidal *function* of van Genuchten (1983) for further analysis:

$$Y = \frac{Y_m}{1 + (c/c_{50})^p} \qquad \ldots (4)$$

where, c_{50} and p are empirical constants. This function also contains three unknown parameters: the maximum yield Y_m, the soil salinity c_{50} at which the yield is declined to 50% of its maximum value, and a parameter p that determines the steepness of the curve (*i.e., the* higher the p, the steeper is the curve). Notice that salinities and yields in eq(4) can be expressed in reduced form. Hence, eq.(4) leads to a dimensionless plot in terms of two scaled variables (relative yield, Y/Y_m and relative concentration, c/c_{50}), and one coefficient, p, reflecting the steepness of the curve.

Figure 2c shows that application of the nonlinear least square inversion program to the barley data resulted in an excellent fit of eq. (4) to the experimental data. In particular, the tailing part at the higher concentrations is now described much better. An excellent fit was also obtained for the Meadow Foxtail data (Fig. 3c). The inverse procedure resulted now in a uniquely defined curve, independent of the initial parameter estimates. These, and many other examples not further reported here, revealed the accuracy and flexibility of eq. (4) in describing a large number of data sets. Encouraged by this success we applied the s-shaped model to all data sets originally used in the analysis of Maas and Hoffman (1977).

Results and Discussion

The data base of Maas and-Hoffman (1977) was divided into four groups: fields crops, forage crops, vegetables and fruit trees. AU fruit tree data sets were discarded because of generally too few or unreliable experimental data. Of the remaining 256 individual data sets, an additional 52 were also judged to be unsuitable for our analysis because of insufficient resolution in the data. Typically, these data sets exhibited severe scattering in the data points, or heavy clustering in only one part of the response curve. The remaining salt tolerance data base consisted of 56 *experiments involving field crops, 80 forage crops, and 68 vegetable crops, giving a total of 204 data sets.

Each data set was analysed for the unknown parameters c_{50}, p and Ym. Table 1 summarizes results for crops for which at least three separate data sets were available. The table lists the means of the estimated c_{50} and p-values, as well as the range in fitted p-values. No clear correlation between p and c_{50} is observed. A statistical analysis of the data showed that p was independent of c_{50}, and hence of the degree of salt tolerance of a particular crop. Also, no statistically significant differences were

Table 1. *Average values for the parameters c_{50} and p in eq. (4) for selected experiment taken from the Maas-Hoffman salt tolerance database*

Crop	Number of data sets	c_{50}		p	
		p variable	p=3	Range	Average
FIELD CROPS					
Barley	7	19.8	20.9	1.82-5.37	3.80
Cotton	8	28.5	28.7	1.55-4.80	3.00
Flax	3	13.4	128	1.28-3.90	2.45
Millet	5	16.6	16.9	1.84-4.30	3.22
Rice	12	6.9	6.9	1.95-8.09	4.54
Sugarcane	9	15.6	16.0	1.66-5.07	2.57
Wheat	5	24.3	23.9	2.70-3.97	3.25
FORAGE CROPS					
Alfalfa	9	18.2	18.4	1.23-7.14	2.51
Barley	3	14.1	14.5	1.92-4.15	2.90
Bermuda	11	27.1	27.6	1.06-6.43	3.16
Clover	11	12.7	13.4	1.51-5.37	2.62
Corn	3	14.8	14.9	1.61-4.03	2.58
Grasses	7	24.4	23.9	1.94-5.57	3.44
Lovegrass	5	16.9	16.8	2.69-5.59	3.85
Wheat grass	12	37.0	37.3	1.51-7.48	3.74
VEGETABLES CROPS					
Bean	11	9.1	9.6	2.06-4.04	2.95
Red beet	3	16.4	16.8	2.18-3.20	2.75
Cabbage	3	11.3	11.6	2.20-3.34	2.58
Carrots	6	8.0	8.0	2.61-5.06	3.14
Corn	4	10.9	10.8	2.42-6.74	4.20
Cucumis	4	14.4	13.9	1.71-5.44	3.49
Lettuce	4	8.9	9.3	2.22-3.71	2.86
Onion	4	8.4	8.6	1.63-8.11	3.75
Peppers	5	10.7	10.3	2.54-4.17	3.34
Radish	3	10.3	10.3	2.38-4.04	3.41
Spinach	3	14.0	14.3	1.55-2.37	1.99
sweet potato	3	8.8	8.7	2.21-9.54	5.07
Tomato	6	12.6	12.1	2.31-8.51	3.88

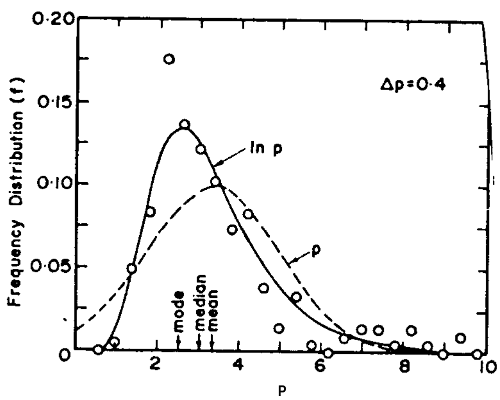

Fig.4. Frequency distribution of 204 fitted p- values

found between the average p-values for the 3 sets of crops. Figure 4 shows a frequency distribution of all 204 fitted p-values, along with the fitted normal and log-normal distributions. Notice that the log-normal frequency distribution fits the data quite well, yielding a mode of 2.55, a median of 3.05 and a mean of 3.34. The variance of the distribution was 2.20. The "average" value of p was judged to be sufficiently close to 3 to allow p to be fixed at that very convenient value. Table 1 shows that the average p-values of many individual crops were also quite close to 3.

Assuming p=3.0; we decided to analyze all 256 experiments again for the two unknown, Y_m and c_{50}. We were especially interested in reanalyzing those experiments which yielded the outliers ***i.e. data sets characterized by very low*** or very high values for p. The results in table 1 indicate p (column 3) and fixed p (column 4). Differences in c50 generally were in the range of 0-5% , and never exceeded 6%.

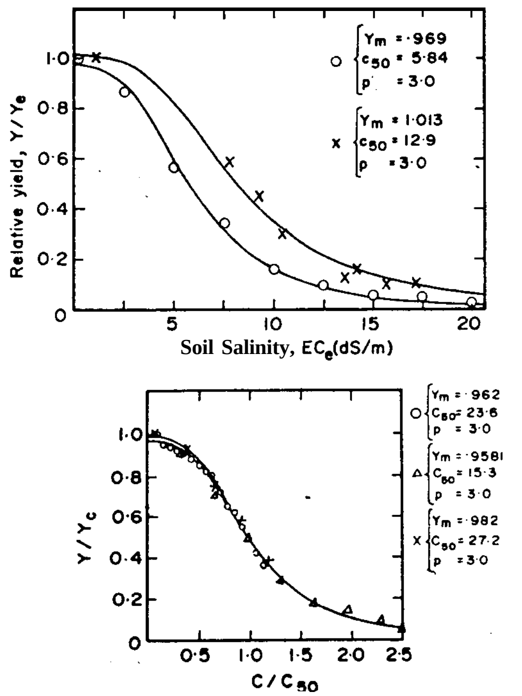

Fig. 5. Observed and fitted salt tolerance curves assuming p = 3 in eq. 4; (a) barley forage – o, meadow foxtail – x. and (b) cotton – o, oat – Δ and rye grass – x

Figure 5 shows typical results obtained when p was fixed at 3. Notice the excellent description in each case, including the experiments for barley (Fig5a) and Meadow foxtail (Fig5b) which were previously analyzed assuming variable p (Figs 2c and 3c, respectively). Figure 6 gives two examples which initially yielded extremely low and high estimate for p. When p was fixed at 3. the shape of the curve for flax (data from Hayward and Spurr. 1944) changed quite significantly. However the residual sum of squares (SSQ) of the optimization [equal to the minimized value of the objective function, Q(b), in eq.(3)] was only little affected by fixing p at 3. Visibly, the more restrictive case of fixed p (Fig. 6b) compares equally well with the data as the variable case (Fig. 6a). The same is not true for the gourd example (data from Paliwal and Maliwal, 1972), which yielded the highest pvalue of all 204 experiments used in our analysis. While the accuracy of the fit definitely decreased (SSQ increased by a factor of about 3). we judge the description to be still quite acceptable ‘for many applications.

The analyses leading to figures 2.3 and 5 show the superiority of the S-shaped model (Eq.4). as compared to the threshold-slope model, in describing most or all salt tolerance data sets. In particular, the optimization process with the S-shaped model was found to be stable and provided statistically unbiased fits of the data. Fixing the value of p at 3.0 only marginally affected the accuracy of the least-square fit for most data sets,

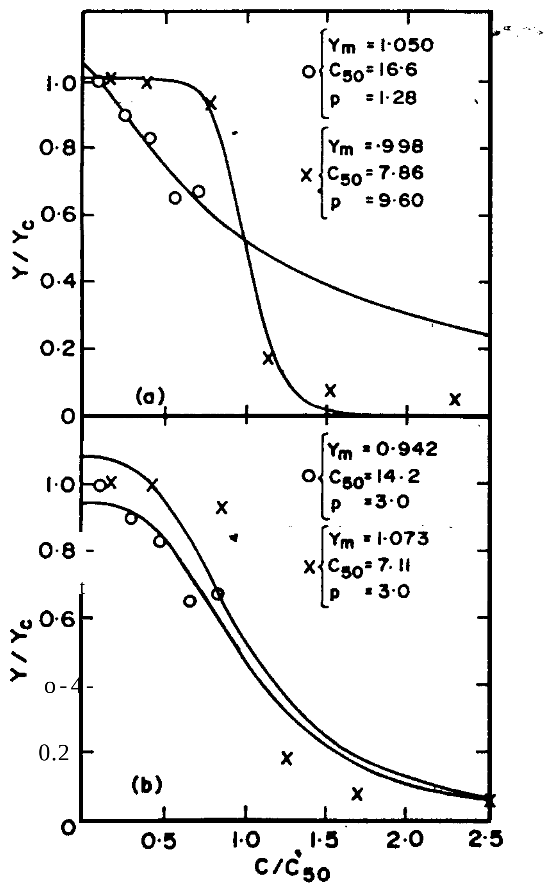

FIg.6. Observed and fitted salt tolerance curves for flax-o and gourd-x (a) with variable p in cq.4. and (b) with fixed p = 3.0 ill eq.4

although a few exceptions were apparent (notably the gourd example in fig. 6). However, most comparisons with p=3 were of the type shown in fig.5. Keeping p constant at 3 results in one unique curve are also evident. Because of fewer unknowns in the response function, the information could be used to more accurately analyze poorly defined or incomplete salt tolerance data sets. This

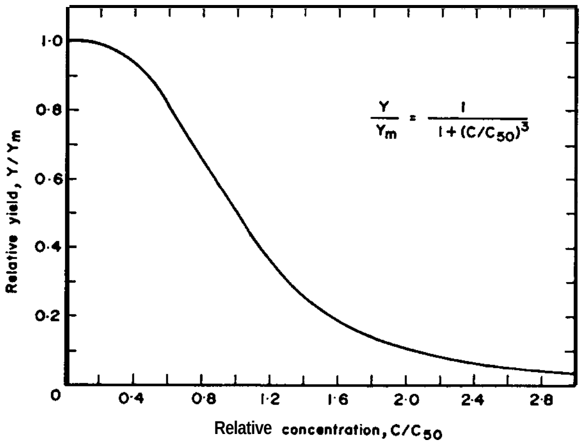

Fig.7. Proposed general dimensionless salt tolerance response function

dimensionless salt tolerance response curve if expressed in terms of the reduced variables c/c_{50} and Y/Y_m (Fig.7). The presence of such an unique curve raises the interesting question whether or not certain similarities exist in the response of different crops to salinity, and perhaps to other yield-limiting stresses as well (notably to water stress). Considerably more work is necessary before these questions can be answered. This additional work must not only involve carefully executed and well-controlled salt tolerance experiments, but probably also requires a statistically more sophisticated analysis of the available data. Our analysis was restricted to the data collected from the literature by Maas and Hoffman in 1977. Numerous experiments have been carried out since that time. We believe that these and other data should be compiled into one computerized database which would be internationally available to both researchers and field practitioners. Such an updated database would be especially useful for phenomenological analyses of the type carried out in this study.

While the presence of a single and unique dimensionless curve may point to some common mechanism governing crop yield response to salinity, other and more immediate applications of point is illustrated in fig. 8 using data for sugar beets (Eaton 1942). This experiment was initially discarded from our analysis because of the severe scattering among the data points. Notice that the data in fig. 8 are also concentrated in only a limited part of the curve, with none of the data points having relative yield of less than 0.5. Clearly, any attempt to analyze this data set should be viewed with caution. However, since a considerable amount of time and labour is generally invested in the experimental determination of these type of salt tolerance data sets, it is also fully understandable that researchers would like to analyze the data, even if the data appear too incomplete for a reliable analysis. The fitted solid line in fig. 8a was obtained when all 3 parameters (Y_m, c_{50} and p) were assumed to be unknowns, resulting in a very high p-value of 15.9, and a maximum, yield Y_m of 0.897 of the control yield, Y_c. When the maximum yield was fixed at the control yield (*i.e., the* yield of the data point having the lowest average soil salinity), the two-parameter analysis produced the solid curve in fig. 8b. Equating Y_m with Y_c may sometimes be an acceptable procedure, especially when the control value is the average of several replicates. Notice that the assumption $Y_m = Y_c$ leads to a radically different p-value (1.45 as compared to 15.9 in fig. 8a); the

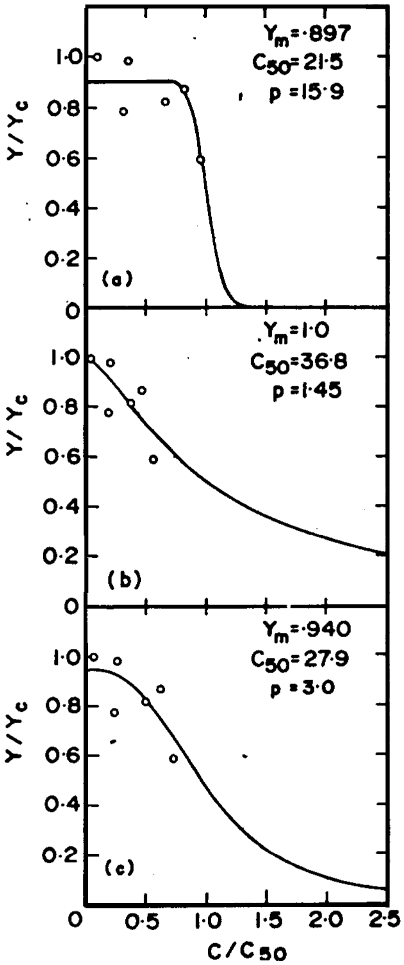

Fig.8 Observed and fitted salt tolerance curves for sugarbeet obtained by fitting q.4 assuming (a) variable Y_m, c_{50} and p; (b) fixed $Y_m = Y_c$ and variable c_{50} and p and (c) p=3 and variable Y_m and c_{50}

value of c_{50} in this case also changed significantly. Fixing p at 3, while keeping Y_m and c_{50} variable, leads to a fit (fig. 8c) which is only slightly less accurate than the complete 3-parameters fit of fig. 8a, but somewhat better than the results of fig. 8b. Visually, the results of fig. 8c appear equally acceptable as those of fig. 8a. Many other examples with similar behaviour could be given. However, like the sugarbeet example, most of those additional data sets were considered to be too incomplete or ill-defined for inclusion in our original analysis involving the 204 salt tolerance data sets.

In conclusion, It can be said that following the early work by Maas and Hoffman (1977). crop salt tolerance is generally described by a piecewise linear model (Eq.1) which assumes no yield decline **until** a "salinity threshold" value, and a linear decrease in yield beyond the threshold . This study shows that an alternative S-shaped response model given by eq.(4) leads to an equally good or better description of experimental data, and with a more stable and unbiased statisdcal fit of the data. Because of fewer unknowns, the dimensionless two-parameter response function may be used to more accurately analyze poorly defined or incomplete salt tolerance data sets. The presence of a unique dimensionless salt tolerance response model may point to some yet undefined common mechanism governing the yield response of crops to salinity, and perhaps to other yield-limiting factors as well (especially to water stress).

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(Received October 1991, Accepted 1993)