Chapter 4. HILLSLOPE SURFACE HYDROLOGY

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4.1 Introduction

The primary purpose of the WEPP surface hydrology component is to provide the erosion component with the duration of rainfall excess, the rainfall intensity during the period of rainfall excess, the runoff volume, and the peak discharge rate. A secondary purpose is to provide the amount of water which infiltrates into the soil for the water balance and crop growth/residue decomposition calculations which are in turn used to update the infiltration, runoff routing, and erosion parameters.

The sequence of calculations relevant to surface hydrology are infiltration, rainfall excess, depression storage, and peak discharge. Infiltration is computed using an implementation of the Green-Ampt Mein-Larson model for unsteady intermittent rainfall. The rainfall excess rate is conceptualized as occurring only when the rainfall rate is greater than the infiltration rate. The volume of rainfall excess is reduced to account for depression storage and runoff is assumed to begin only after the depression storage has been satisfied. For runoff events which produce a partial equilibrium hydrograph, the rainfall excess volume is reduced to account for water infiltrating during the recession of the event. Two methods are used to compute the peak discharge rate depending if the model is run in a continuous or single storm mode and if there are multiple overland flow elements (OFE). A semi-analytical solution of the kinematic wave model is used to compute the runoff hydrograph when the model is run in the single storm mode for a single OFE or when multiple OFEs can be treated as a single OFE. A peak discharge approximation based on the kinematic wave model is used for most events when the model is run in the continuous simulation mode. Infiltration and rainfall excess on multiple OFEs are approximated by either averaging the infiltration parameters and treating the multiple OFEs as a single OFE or by computing a simple water balance to determine if runoff occurs. For multiple OFEs, an equivalent depth-discharge coefficient for the kinematic wave model is computed based on the equilibrium storage of water on a cascade of OFEs.

This chapter describes the processes and equations for a single OFE in sections 4.2 to 4.4, for multiple OFEs in section 4.5, and some limitations of the methodologies in section 4.6.

4.2 Infiltration

The average infiltration rate, f_i ($m \cdot s^{-1}$), for an interval is computed as

$$f_i = \frac{F_i - F_{i-1}}{t_i - t_{i-1}} \qquad [4.2.1]$$

where the subscripts i and $i-1$ refer to the current and previous time intervals respectively, F = cumulative infiltration depth (m), and t = time (s). The cumulative infiltration is computed using the Green-Ampt Mein-Larson (GAML) model (Mein and Larson, 1973) as presented by Chu (1978) for the case of unsteady rainfall and multiple times to ponding.

Chu (1978) computed an indicator, C_u (m), that determines if ponding occurs within a given interval of rainfall intensity given that there was no ponding at the beginning of the interval as

$$C_u = R_i - V_i - \left(\frac{K_e \ \Psi \ \theta_d}{r_{i-1} - K_e} \right) \qquad [4.2.2]$$

where R is the cumulative rainfall depth (m), V is the cumulative rainfall excess depth (m), K_e = effective saturated hydraulic conductivity ($m \cdot s^{-1}$), Ψ = average capillary potential (m), θ_d = soil moisture deficit ($m \cdot m^{-1}$), and r is the rainfall rate ($m \cdot s^{-1}$). The soil moisture deficit is computed as

$$\theta_d = \eta_e - \theta_v \qquad [4.2.3]$$

where η_e = effective porosity ($m \cdot m^{-1}$) and θ_v = initial volumetric water content ($m \cdot m^{-1}$). The cumulative rainfall excess depth is computed as

$$V_i = R_i - F_i \qquad [4.2.4]$$

If C_u is positive, ponding occurs before the end of the interval; if it is negative, no ponding occurs. The time to ponding, t_p (s), is computed as

$$t_p = \left(\frac{K_e \ \Psi \ \theta_d}{r_{i-1} - K_e} - R_{i-1} + V_{i-1} \right) \frac{1}{r_{i-1}} + t_{i-1} \qquad [4.2.5]$$

When there is ponding within a rainfall interval, the cumulative infiltration depth is computed using

$$K_e \ t_c = F_i - \Psi \ \theta_d \ \ln \left[1 + \frac{F_i}{\Psi \ \theta_d} \right] \qquad [4.2.6]$$

in a Newton-Raphson iteration. The time, t_c, in Eq. 4.2.6 is corrected to account for the difference between instantaneous time to ponding and the actual time to ponding and is computed as

$$t_c = t_i + \frac{R_{tp} - V_{i-1} - \Psi \ \theta_d \ \ln \left(1 + \frac{R_{tp} - V_{i-1}}{\Psi \ \theta_d} \right)}{K_e} - t_p \qquad [4.2.7]$$

where R_{tp} = amount of cumulative rainfall (m) at the time to ponding.

The indicator for the end of ponding C_p during an interval, assuming the surface was ponded at the beginning of the interval is

$$C_p = R_i - F_i - V_i \qquad [4.2.8]$$

If C_p is positive, ponding continues, if it is negative ponding ceases within the interval. When there is no ponding within an interval, the cumulative infiltration is computed as

$$F_1 = R_i - V_{i-1} \qquad [4.2.9]$$

4.3 Rainfall Excess

Rainfall excess is the portion of the rainfall which ponds on the surface during the period when the rainfall rate exceeds the infiltration capacity. The time intensity distribution of rainfall excess is transformed into a time intensity distribution of runoff (the hydrograph) by the kinematic wave model or a value of peak discharge by the approximate method described in the following sections. However, before the rainfall excess rate is computed, the volume is adjusted for soil saturated conditions and depression storage. The rainfall excess volume is computed in conjunction with the infiltration calculations as

$$\begin{aligned} V_i &= R_i - F_i && \text{for} \quad r_i > f_i \quad \text{and} \quad F_i < S_p \\ V_i &= V_{i-1} && \text{for} \quad r_i \le f_i \quad \text{and} \quad F_i < S_p \\ V_i &= R_i && \text{for} \quad F_i \ge S_p \end{aligned} \qquad [4.3.1]$$

where S_p = the upper limit of water storage (m) in the top two soil layers. The storage upper limit is computed as

$$S_p = K_{\min}\, D_r + \max\left(O,\ \sum_{j=1}^{2} (UL_j - ST_j) \right) \qquad [4.3.2]$$

where $K_{\min}$ = minimum saturated hydraulic conductivity of the two layers ($m{\cdot}s^{-1}$), D_r = duration of rainfall (s), UL_j = upper limit of soil moisture storage (m), and ST_j = current soil moisture storage (m). If the cumulative infiltration during a rainfall event exceeds S_p, then the remainder of the rainfall becomes rainfall excess.

The rainfall excess volume computed by Eq. 4.3.1 is reduced for the amount of depression storage which is defined as the portion of rainfall excess which is held in storage caused by micro-variations in topography and which eventually infiltrates into the soil. Depending on the degree of microrelief occurring on a surface, depression storage can greatly impact runoff amounts and rates. The adjustment for depression storage is computed after the infiltration calculations as

$$\begin{aligned} V_i &= O && \text{for} \quad V_i \le S_d \\ V_i &= V_i - S_d && \text{for} \quad V_i > S_d \end{aligned} \qquad [4.3.3]$$

where S_d = maximum depression storage (m). Note that depression storage is assumed to be filled before runoff begins. The maximum depression storage is computed from a relationship derived by Onstad (1984) from an analysis of over 1000 tilled plots. This relationship is

$$S_d = 0.112\, r_r + 3.1\, r_r^2 - 1.2\, r_r\, S_o \qquad [4.3.4]$$

where r_r is the random roughness (m) and S_o is the slope of the flow surface ($m{\cdot}m^{-1}$). The term "maximum" is used because there are cases, generally for small runoff events, when depression storage will not be completely satisfied.

The three basic conditions considered by the depression storage component depend on the number of discrete rainfall excess bursts, if potential depression storage is completely satisfied, and if the amount of water held in depression storage infiltrates before the next burst of rainfall excess. The first condition is when depression storage is completely satisfied. At the end of the event when the rainfall excess rate is zero, the water held in depression storage is added to the cumulative infiltration volume. The second condition is when the rainfall excess volume is less than the potential depression storage volume. In this case the total rainfall excess volume is added to the cumulative infiltration volume and total runoff equals zero. The third condition is when there is a hiatus of rainfall excess. The water held in depression storage is infiltrated at the infiltration rate which occurred at the end of the last burst of rainfall excess. If the next burst of rainfall excess occurs before all the depression storage water is infiltrated then the remainder becomes part of the potential depression storage.

After the above adjustments are made to the rainfall excess volume, an average rainfall excess rate, v_i ($m \cdot s^{-1}$), for an interval is computed as

$$v_i = \frac{V_i - V_{i-1}}{t_i - t_{i-1}} \quad [4.3.5]$$

4.4 Peak Discharge

Dynamic infiltration-hydrograph models for overland flow consist of an infiltration function which computes the infiltration rate as it varies with time from an unsteady rainfall input and a routing function which transforms rainfall excess into flow depths on a flow surface. The choice of the infiltration function is somewhat arbitrary, but the routing function is generally some form of the St. Vennant shallow water equations. One such form, the kinematic wave model, has been shown (Woolhiser and Liggett, 1967) to be a valid approximation for most overland flow cases.

WEPP uses two methods of computing the peak discharge; a semi-analytical solution of the kinematic wave model (Stone et al., 1992) and an approximation of the kinematic wave model. The first method is used when WEPP is run in a single event mode while the second is used when WEPP is run in a continuous simulation mode.

4.4.1 Kinematic wave model

The kinematic equations for flow on a plane are the continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = v \quad [4.4.1]$$

and a depth-discharge relationship

$$q = \alpha\, h^m \quad [4.4.2]$$

where h = depth of flow (m), q = discharge per unit width of the plane ($m^3 \cdot m^{-1} \cdot s^{-1}$), α = depth-discharge coefficient, m = depth-discharge exponent, and x = distance from top of plane (m). The Chezy relationship is used for overland flow routing in WEPP so $\alpha = C\, S_o^{0.5}$ where C = Chezy coefficient ($m^{0.5} \cdot s^{-1}$) and m = 1.5. The initial and boundary conditions are

$$h(x,0) = h(0,t) = 0 \quad [4.4.3]$$

Eqs. 4.4.1 and 4.4.2 given Eq. 4.4.3 are solved using the method of characteristics. The method involves rewriting Eqs. 4.4.1 and 4.4.2 as simple ordinary differential equations on characteristic curves in the t-x plane. The equations for depth and distance along a characteristic $c\,(t,x)$ at a given time are (see Eagleson, 1970, for a derivation of the characteristic equations)

$$\frac{dh}{dt} = v(t) \quad [4.4.4]$$

and

$$\frac{dx}{dt} = \alpha\, m\, h(t)^{m-1} \quad [4.4.5]$$

The characteristic, Eq. 4.4.5, defines a locus of points in the time-space plane on which the flow depth is computed by Eq. 4.4.4. The characteristic equations are integrated to get

$$h = h_1 + \int_{t_1}^{t_2} v(w)\, dw \tag{4.4.6}$$

$$x = x_1 + \alpha\, m \int_{t_1}^{t_2} h(w)^{m-1}\, dw \tag{4.4.7}$$

where x_1 = distance (m) on a characteristic at time t_i, h_1 = depth (m) on a characteristic at time t_1, t_1 and t_2 = limits of integration (s), and w = dummy variable of integration.

The general solution procedure is to solve Eq. 4.4.6 for the depth of flow at a time and then solve Eq. 4.4.7 for the distance from the top of the plane that the depth occurs. Because we are interested in the hydrograph at the end of the plane, the distance solved for in Eq. 4.4.7 is the length of the plane. The discharge rate is computed by solving Eq. 4.4.2 given the depth found by Eq. 4.4.6. Because rainfall excess is conceptualized by Eq. 4.3.1, the hydrograph generated by Eqs. 4.4.6 and 4.4.7 exhibit two physically unrealistic properties; partial equilibrium or flat topped hydrographs and infinite runoff duration.

Partial equilibrium hydrographs occur when the rainfall excess ends before the c(0,0) characteristic reaches the end of the plane. The c(0,0) characteristic is that which originates at the top of the plane at the start of rainfall excess (also termed the equilibrium characteristic because it denotes the time at which the plane reaches steady state flow under constant rainfall excess). The solution of Eqs. 4.4.6 and 4.4.7 for partial equilibrium case are

$$h(t) = \int_{0}^{t_z} v(w)\, dw \equiv h_z \tag{4.4.8}$$

$$x = x_z + \alpha\, m\, h_z^{m-1}\, (t - t_z) \tag{4.4.9}$$

where t_z = time rainfall excess ends (s), h_z = depth on the characteristic (m) at times $\geq t_z$, and x_z = location on the plane of h_z. Note that by Eq. 4.4.8, the flow depth is a constant and that by Eq. 4.4.9, the characteristic is a straight line. Partial equilibrium is physically unrealistic on an infiltrating surface because at the time the infiltration rate is greater than the rainfall rate or rainfall ceases, infiltration continues as long as water is ponded on the surface. Thus the runoff flow depth immediately begins to decrease. The definition of rainfall excess (Eq. 4.3.1) does not allow for infiltration after rainfall ceases, so partial equilibrium hydrographs occur. It is difficult to generalize if a particular application will result in partial equilibrium if the rainfall excess is computed using the Green and Ampt equation with unsteady rainfall input. However, for the case of constant rainfall excess, the time to kinematic equilibrium, t_e (s), which is the time that c(0,0) reaches the end of the plane is

$$t_e = \left(\frac{L}{\alpha\, v^{m-1}} \right)^{1/m} \tag{4.4.10}$$

where L = length of the overland flow surface (m). As can be seen from Eq. 4.4.10, the longer the plane, the smaller the rainfall excess rate, or the rougher the surface, the longer it takes c(0,0) to reach the end of the plane. For example, on a rainfall simulator plot with a length of 10.7 meters, a slope of 5%, a Chezy roughness coefficient of 2, and a rainfall excess rate of 10 $mm \cdot h^{-1}$, the time to kinematic equilibrium is 9.8 minutes. Thus, if the duration of rainfall excess is less than 9.8 minutes, the hydrograph will be in partial equilibrium. If the duration is greater partial equilibrium will not occur.

The recession of the hydrograph is characterized by the water surface elevation decreasing everywhere on the plane. The solution of Eqs. 4.4.6 and 4.4.7 for this case are

$$h(t) = \int_{t_o}^{t_z} v(w)\, dw \equiv h_r \qquad [4.4.11]$$

$$L = x_z + \alpha\, m\, h_r^{m-1}\, (t - t_z) \qquad [4.4.12]$$

where t_0 = time (s) the characteristic originated at the top of the plane, h_r = depth (m) on the characteristic, and x_z = location on the plane of h_r. By Eq. 4.4.11, the depth is constant on a characteristic and, by Eq. 4.4.12, time approaches infinity as the depth approaches zero. As a result, routing of the rainfall excess in WEPP is terminated when either the routed runoff volume equals 95% of the rainfall excess volume or the discharge rate is 10% of the peak discharge rate, whichever occurs first.

4.4.2 Approximate method of calculating peak flow

As mentioned in the introduction, the surface hydrology component supplies the erosion component with the volume and peak rate of runoff and the duration of rainfall excess. Even though the semi-analytical solution of the kinematic wave model for overland flow described in Section 4.4.1 executes rapidly, an approximation was developed to further decrease the program execution time. The basis of the approximation is a relationship among the time to kinematic equilibrium, the duration of rainfall excess, D_v (s), the peak rainfall excess rate, v_p ($m \cdot s^{-1}$), and the average rainfall excess rate, v_a ($m \cdot s^{-1}$).

4.4.2.1 Constant rainfall excess

For constant rainfall excess, v_a ($m \cdot s^{-1}$), the flow depth and discharge rate increases during the period $t < t_e$ and is constant for $t \geq t_e$. If the duration of the rainfall excess is less than t_e, then the maximum flow depth, h_p (m), is

$$h_p = v_c\, D_v \qquad [4.4.13]$$

Substituting Eq. 4.4.13 into Eq. 4.4.2, using the definition of t_e, and simplifying, the peak discharge, q_p ($m \cdot s^{-1}$), is

$$q_p = v_c \left(\frac{D_v}{t_e} \right)^m \quad \text{for} \quad D_v < t_e \qquad [4.4.14]$$

When the duration of rainfall excess is greater than the time to kinematic equilibrium (i.e. equilibrium), then the peak flow rate is simply

$$q_p = v_c \qquad \text{for} \quad D_v > t_e \qquad [4.4.15]$$

Eqs. 4.4.14 and 4.4.15 can be generalized by defining the following dimensionless quantities as

$$q_* = \frac{q_p}{v_c}$$
$$t_* = \frac{t_e}{D_v} \qquad [4.4.16]$$

so that Eqs. 4.4.14 and 4.4.15 become

$$q_* = t_*^{-m} \qquad \text{for} \quad t_* \geq 1 \qquad [4.4.17]$$

$$q_* = 1 \qquad \text{for} \quad t_* < 1 \qquad [4.4.18]$$

Eqs. 4.4.17 and 4.4.18 are illustrated in Figure 4.4.1.

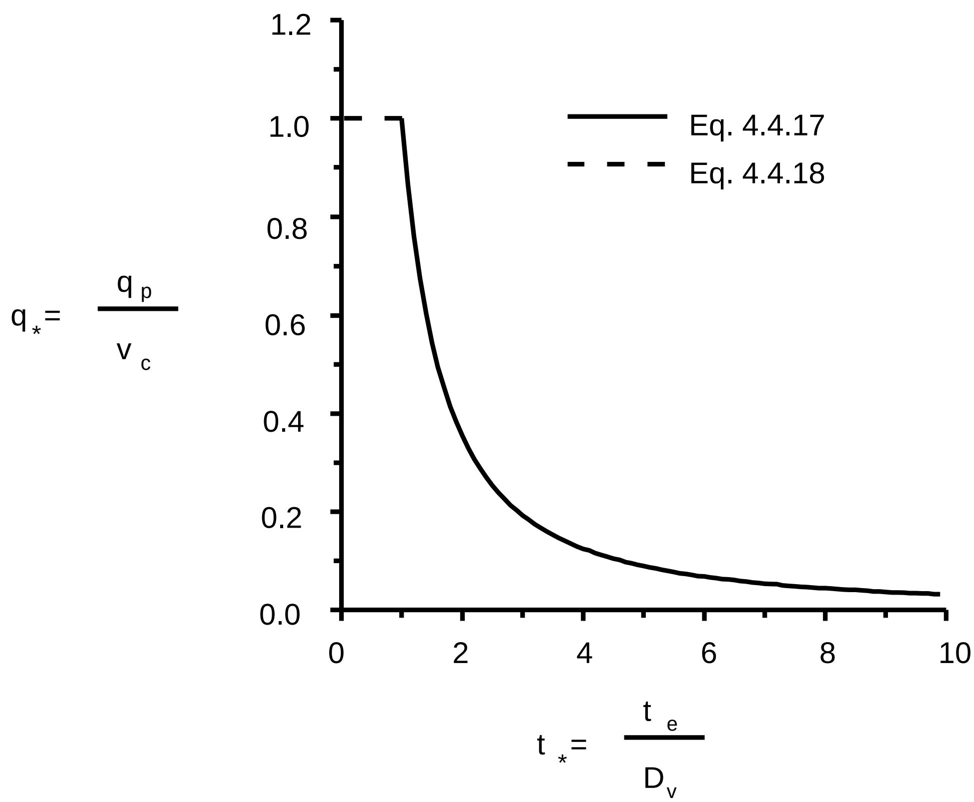

Figure 4.4.1. Relationship between t_* and q_* to compute peak discharge for constant rainfall excess.

4.4.2.2 Variable rainfall excess

For variable rainfall excess, the definition of t_e (Eq. 4.4.10) is not exactly true. In addition, for times greater than when the characteristic from time and distance zero reaches the end of the OFE, the discharge rate approaches but never exactly equals the rainfall excess rate. Using Eqs. 4.4.17 and 4.4.18 as a starting point, first define the average rainfall excess rate as

$$\nu_a = \frac{V_t}{D_\nu} \qquad [4.4.19]$$

where V_t = total rainfall excess depth (m). Using Eq. 4.4.19, compute the time to equilibrium, t_a as

$$t_a = \left(\frac{L}{\alpha \, \nu_a^{m-1}} \right)^{1/m} \qquad [4.4.20]$$

Then re-define the dimensionless quantities in Eq. 4.4.16 as

$$q_* = \frac{q_p}{v_a}$$
$$t_* = \frac{t_a}{D_v} \qquad [4.4.21]$$

and define a dimensionless rainfall excess rate as

$$\nu_* = \frac{\nu_a}{\nu_p} \qquad [4.4.22]$$

The WEPP model was run using the kinematic routing procedure to produce values of q_*, t_*, and v_* for a range of rainfall characteristics, soil types, initial conditions, and depth-discharge coefficients (Table 4.4.1). Inspection of the results of the simulations plotted in Figure 4.4.2 suggested that the following relatively simple equations could be used to approximate the peak discharge

$$q_* = t_*^{-m} \qquad \text{for} \quad t_* \geq 1 \qquad [4.4.23]$$

$$q_* = \frac{1}{t_*} \qquad \text{for} \quad 1 > t_* \geq t_{**} \qquad [4.4.24]$$

$$q_* = \frac{1}{\nu_*} - .6 \, \frac{1 - \nu_*}{\nu_*} t_* \qquad \text{for} \quad t_{**} > t_* \geq 0 \qquad [4.4.25]$$

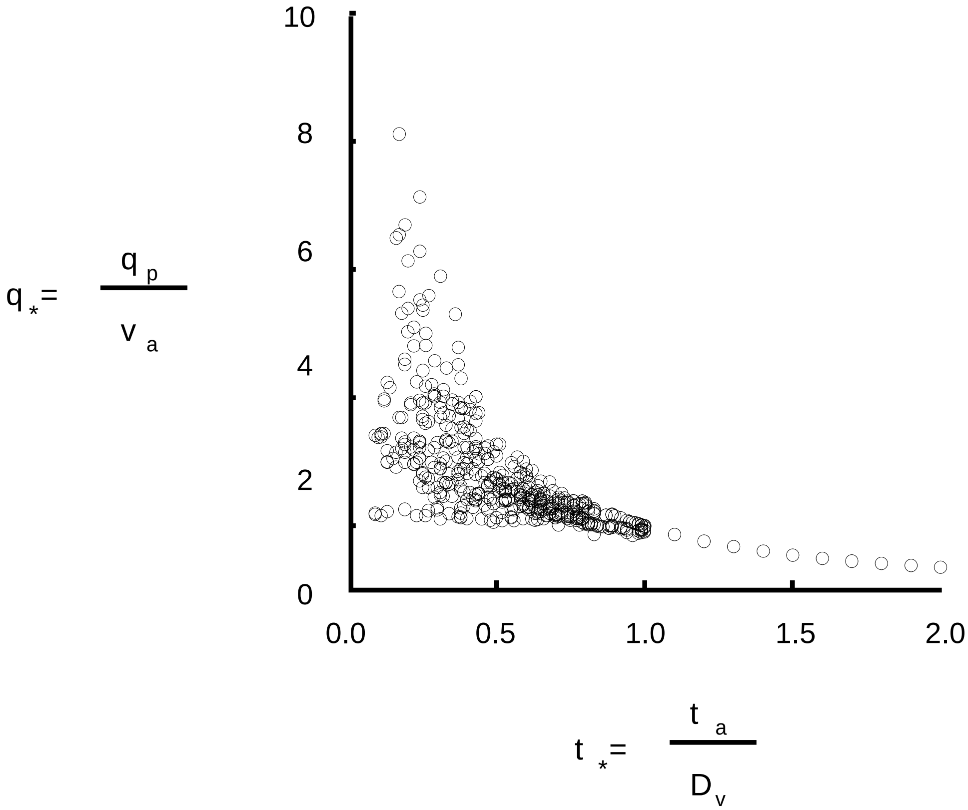

Figure 4.4.2. t_* versus q_* generated by the WEPP model for variable rainfall excess.

Table 4.4.1. Range of t_*, V_* and q_* used to determine Eqs. 4.4.23, 4.4.24 and 4.4.25.

Variable	**Minimum**	**Maximum**
t_*	.09	10.0
v_*	.08	1.0
q_*	.07	8.0

The intersection of Eq. 4.4.24 and 4.4.25, t_{**}, is found by combining the equations, substituting t_{**} for t_*, and solving for t_{**} using the quadratic formula as

$$t_{**} = \frac{1 - \sqrt{1 - 2.4\,(\nu_* - \nu_*^2)}}{1.2\,(1 - \nu_*)} \qquad [4.4.26]$$

Note that the coefficient, .6, in the slope term of Eq. 4.4.25 will change if the Manning relationship is used to compute α. The relationships of Eqs. 4.4.23, 4.4.24 and 4.4.25 are plotted in Figure 4.4.3. The average error between the peak discharge computed using the kinematic wave model and the above equations using the range of conditions for the simulation was 1%, 10%, and 5% for Eqs. 4.4.23, 4.4.24 and 4.4.25 respectively or a total combined error of 6.6%.

The approximate method was tested by comparing the peak discharge computed by the equations and that computed by the kinematic routing procedure using the rainfall disaggregation validation data set presented in Chapter 2. Referring to the top of Figure 4.4.4, when the observed rainfall distribution is used, the approximate method agrees closely with the kinematic routing procedure except for one event. When the disaggregated rainfall distribution is used the approximate method agrees very closely with the kinematic routing procedure.

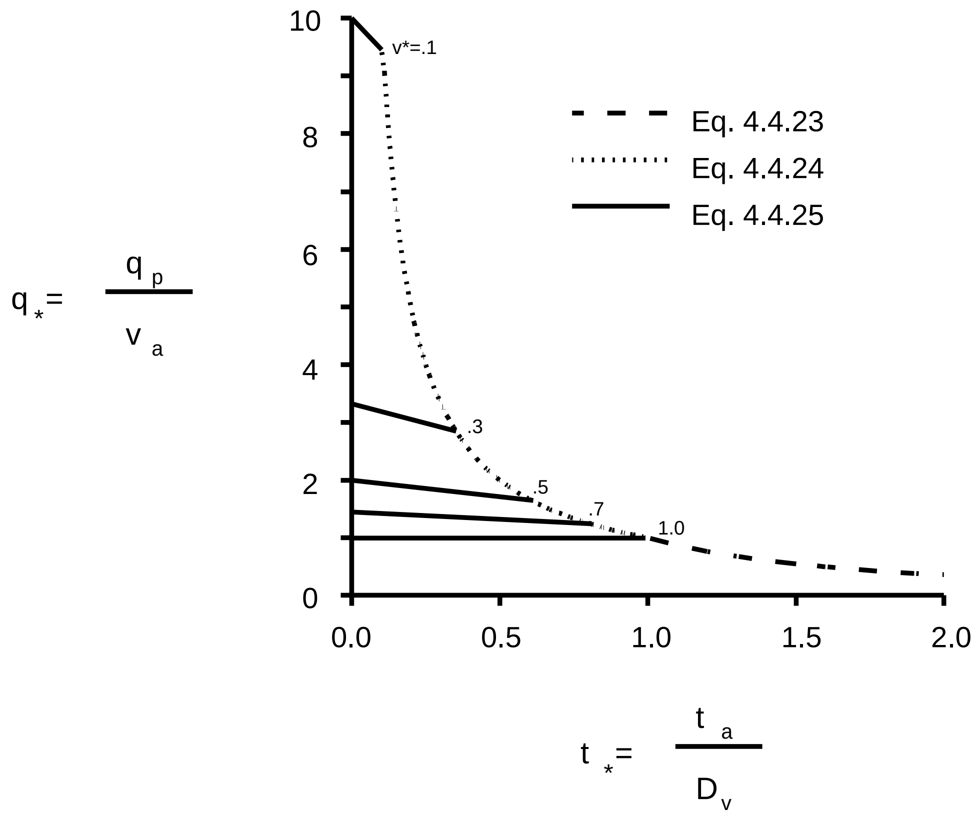

Figure 4.4.3. Relationship between t_* and q_* to compute peak discharge for variable rainfall excess.

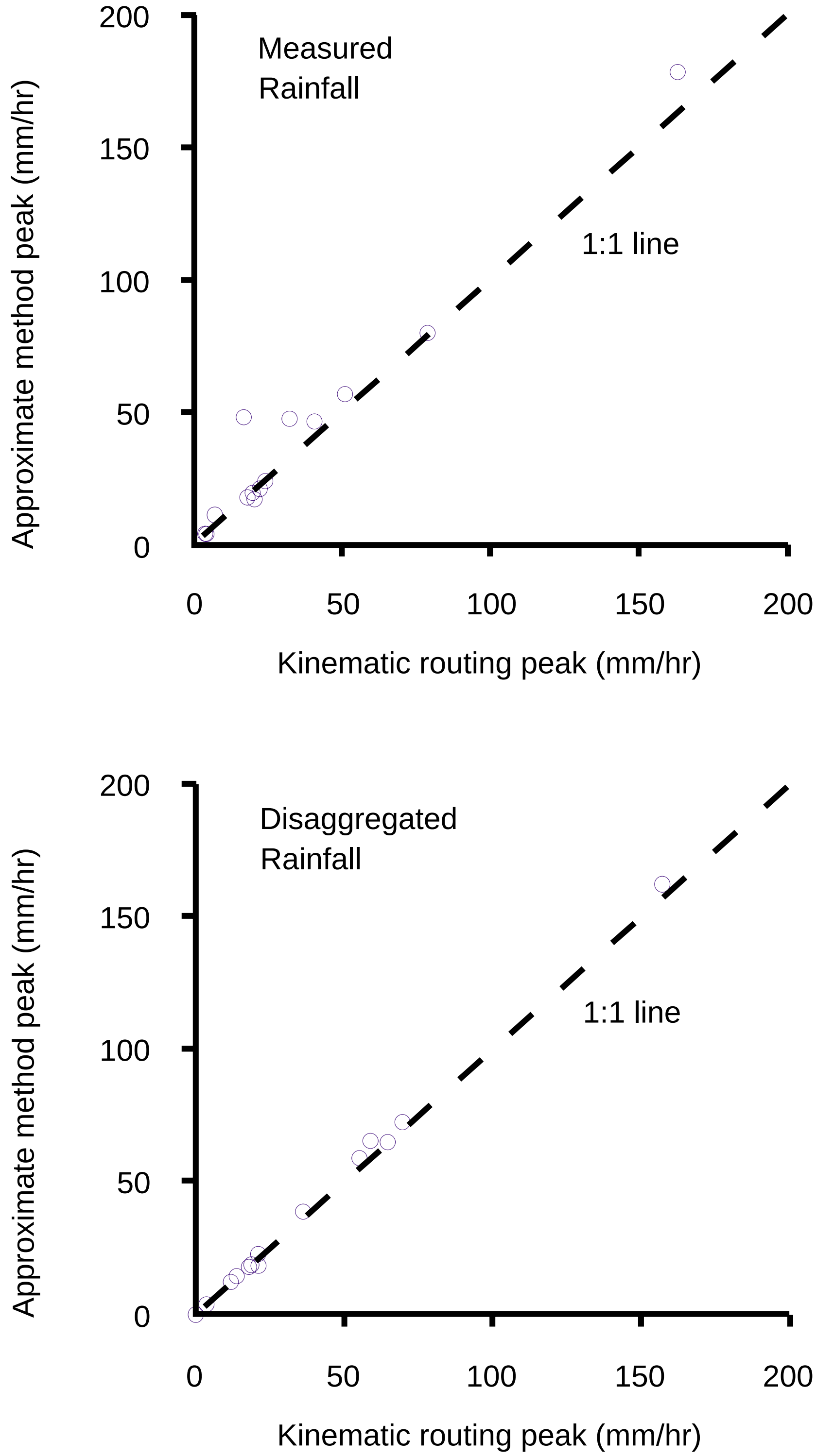

Figure 4.4.4. Kinematic peak discharge versus approximate method peak discharge for the disaggregation validation data set using (top) measured rainfall and (bottom) disaggregated rainfall input.

4.4.3 Recession infiltration

For the case of partial equilibrium previously described in section 4.4.1, the runoff volume can be significantly less than the rainfall excess volume. The reason is that there is no interaction between the routing equations and the infiltration equations so that in effect, the rainfall excess is routed as if the flow surface was impermeable. To account for the infiltration during the recession of the hydrograph, WEPP uses a relationship among the final infiltration rate, average rainfall excess rate, and total rainfall excess volume. By defining the following dimensionless quantities

$$Q_* = \frac{Q_\nu}{V_t} \qquad f_* = \frac{f_f}{\nu_a} \tag{4.4.27}$$

where Q_ν = adjusted runoff depth (m) and f_f = final infiltration rate ($m \cdot s^{-1}$) at the last time of non-zero rainfall excess rate, the reduction in runoff volume is computed as (Stone et al., 1993)

$$Q_* = \frac{1}{m+1}\,\frac{f_*+1}{f_*}t_*^{-m} \qquad \text{for } t_* \geq \left(\frac{f_*+1}{f_*}\right)^{1/m} \tag{4.4.28}$$

and

$$Q_* = 1 - \frac{m}{m+1}\left(\frac{f_*}{f_*+1}\right)^{1/m} t_* \qquad \text{for } t_* < \left(\frac{f_*+1}{f_*}\right)^{1/m} \tag{4.4.29}$$

Eqs. 4.4.28 and 4.4.29 are illustrated in Figure 4.4.5 for the depth-discharge coefficient computed using the Chezy coefficient.

4.4.4 Runoff duration

The WEPP Model uses the steady-state sediment continuity equation as a basis for erosion computations (Chapter 11). As a result, the steady-state runoff discharge rate is assumed to be the peak discharge rate computed by the kinematic routing procedure or the approximate method described above. Under this assumption, the computed runoff duration would not maintain continuity between the peak discharge rate and the runoff volume. In order to maintain continuity, an effective duration, D_e (s), is computed in the WEPP surface hydrology component and is used in the rill erosion computations:

$$D_e = \frac{Q_v}{q_p} \tag{4.4.30}$$

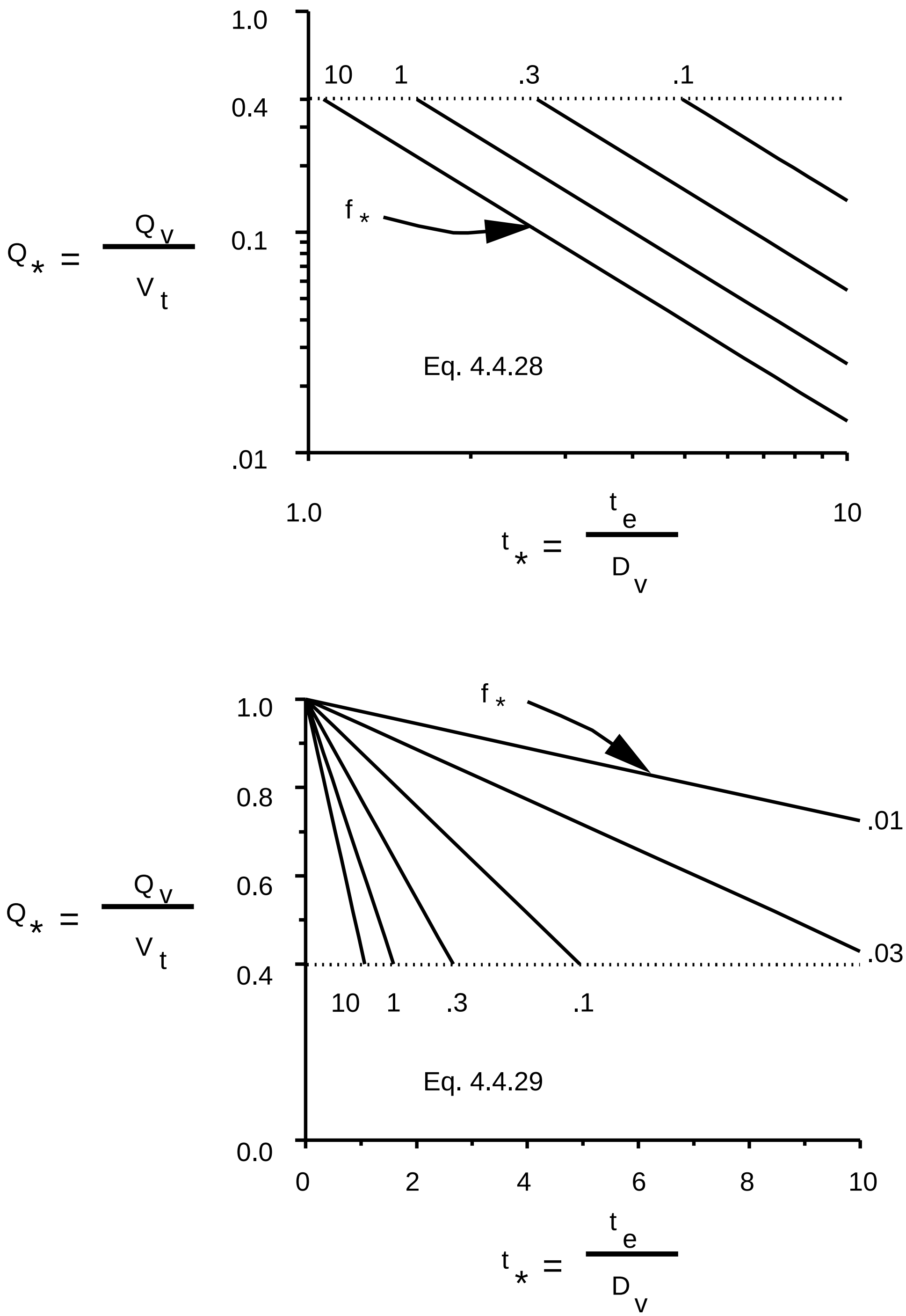

Figure 4.4.5. Relationship between t_* and Q_* for computing the reduction of rainfall excess volume due to recession infiltration.

4.5 Multiple Overland Flow Elements

The preceding sections described how infiltration, rainfall excess, depression storage, and peak discharge are calculated on a single overland flow element (OFE) or plane. Note that the rainfall excess is calculated first and then used as input to the overland flow routing routines in order to avoid a numerical solution to the kinematic wave equation. Although this simplification has the advantage of speeding up the computational time, it does not allow for any interaction between infiltration and runoff. Once rainfall ends, infiltration and rainfall excess ends. Under most situations, the approximation works well for a single OFE; however calculating infiltration and runoff on multiple OFEs with different infiltration rates requires a modification of the method.

4.5.1 Problem statement

For those cases where soil properties or vegetation characteristics (i.e. strip cropping) vary downslope, the hillslope can be divided into multiple overland flow elements (OFEs) oriented along the contour of the hillslope to account for differences in infiltration, runoff, and/or erosion parameters. Because of these differences, the hydrologic response of each individual OFE will be different from that of the OFE immediately above and below. For example, there will be times when an OFE will produce no rainfall excess for a given rainfall event while the OFE above will produce rainfall excess for the same rainfall event.

There are four general cases which can arise on any OFE that has a OFE above it depending on if the current OFE has rainfall excess and if the upper OFE has runoff. Herein we use the definition of rainfall excess given by Eq. 4.3.1, that is water ponding on the surface when the rainfall rate is greater than the infiltration rate. In the following, Q_{j-1} = runoff from the upper OFE, V_j = rainfall excess from the current OFE, and Q_j = runoff from the current OFE, and the subscript j is the index of the OFE.

Case one: $Q_{j-1} = 0$, $V_j = 0$, $Q_j = 0$. The first case occurs when both runoff from the upper OFE and rainfall excess from the current OFE are zero; thus the runoff from the current OFE is also zero.

Case two: $Q_{j-1} \geq 0$, $V_j > 0$, $Q_j > 0$. The second case occurs when the upper OFE has runoff greater or equal to zero and the current OFE has rainfall excess greater than zero.

Case three: $Q_{j-1} > 0$, $V_j = 0$, $Q_j > 0$. The third case occurs when the upper OFE has runoff and the current OFE has zero rainfall excess. In this case the runoff from the upper OFE runs on to the current OFE and exceeds the infiltration capacity of the current OFE so that there is net runoff from the current OFE.

Case four: $Q_{j-1} > 0$, $V_j = 0$, $Q_j = 0$. The fourth case is similar to case three except the amount of runon from the upper OFE does not exceed the infiltration capacity of the current OFE and there is no runoff from the current OFE.

Although the peak discharge computations of WEPP can be applied for case 2 using the concept of the equivalent plane described later in this chapter, a numerical method (for example the KINEROS model, Woolhiser et al., 1990) is generally used to compute the hydrograph for the runon-runoff cases of case three and four. However, in order to avoid the numerical solutions, a method was developed to approximate the runon-runoff cases.

4.5.2 Infiltration

To determine which of the four cases described above apply, a weighted average saturated hydraulic conductivity, K_a, and matric potential, $(\Psi\ \theta_d)$a, are calculated for the OFEs under consideration as

$$K_a = \frac{\sum_{j=p}^{n} K_{ej}\, x_j}{\sum_{j=p}^{n} x_j} \qquad [4.5.1]$$

$$(\Psi\, \theta_d)_a = \frac{\sum_{j=p}^{n} \Psi_j\, \theta_{dj}\, x_j}{\sum_{j=p}^{n} x_j} \qquad [4.5.2]$$

where K_{ej} and $\Psi_j\, \theta_{dj}$ = the effective hydraulic conductivity and matric potential for the *jth* OFE respectively, p = the first OFE above the current OFE that has non-zero runoff, and n = the current OFE. These average parameters are used to compute infiltration as described in section 4.2.

4.5.3 Rainfall excess

If the rainfall excess computed using the average GAML parameters is greater than zero, then it is a case two condition, and the rainfall excess volume and rates are computed as for a single OFE. For example, consider a cascade of 10 OFEs and let the bottom most OFE or the 10th OFE be the OFE being processed. If all the OFEs above have non-zero runoff, then the averages computed by Eqs. 4.5.1 and 4.5.2 will be the average for the ten OFEs. If, however a OFE has had zero runoff, say, the third OFE, then the average will be the average for OFEs 4 through 10. Depression storage for case two OFEs is computed using an average maximum depression storage as

$$S_a = \frac{\sum_{j=p}^{n} S_{dj}\, x_j}{\sum_{j=p}^{n} x_j} \qquad [4.5.3]$$

and used in the same manner as for a single OFE.

If the rainfall excess computed is equal to zero, then it is a case three or four condition and a potential infiltration volume capacity per unit width, F_p $(m^3 \cdot m^{-1})$, is calculated and compared to the unit width volume of water, F_h $(m^3 \cdot m^{-1})$ entering the current OFE from the upper OFE. The potential infiltration capacity is computed using a Taylor Series expansion approximation of the GAML model (Stone et al., 1994) and the maximum depression storage, $(S_d)_j$, for the current OFE

$$F_p = \left[\left(t_{f*} + (2\, t_{f*})^{1/2} - 0.2987\, t_{f*}^{0.7913} \right) (\Psi\, \theta_d)_j + (S_d)_j \right] L_j \qquad [4.5.4]$$

where

$$t_{f*} = \frac{(K_e)_j\, D_r}{(\Psi\, \theta_d)_j} \qquad [4.5.5]$$

The volume (depth) of water entering the OFE is the runon, Q_{j-1} (m), from the upper OFE and the rainfall, R_j (m), on the current OFE or

$$F_h = Q_{j-1}\ (L_e)_{j-1} + R_j\ L_j \tag{4.5.6}$$

where $(L_e)_{j-1}$ = length (m) of all the OFEs above the current OFE which have runoff and L_j = length (m) of the current OFE. Case three occurs when

$$F_h - F_p > 0 \tag{4.5.7}$$

The rainfall excess depth is computed as

$$V_j = \frac{F_h - F_p}{(L_e)_j} \tag{4.5.8}$$

and the maximum rainfall excess rate is

$$(\nu_p)_j = (\nu_p)_{j-1}\ \frac{V_j}{V_{j-1}} \tag{4.5.9}$$

Case four occurs when

$$F_h - F_p \leq 0 \tag{4.5.10}$$

and the runoff excess depth is zero.

4.5.4 Peak discharge

For the case of multiple OFEs, the characteristic solution presented in section 4.5.1 produces what are termed kinematic shock waves. These are simply locations within the t-x plane where two characteristics intersect. At the point of intersection, the depth on each characteristic is different, thus introducing a discontinuity in the flow depth. This situation arises when an upper OFE has a depth-discharge coefficient and/or a rainfall excess rate which is larger than the lower OFE. Examples of when kinematic shocks will occur are water flowing on an initially unponded surface as is the case with surface irrigation or runoff flowing from a less permeable soil onto a more permeable, ponded soil.

Although methods exist which avoid kinematic shock formation in their solution (for example, see KINEROS, Woolhiser et al., 1990), their execution time is greater than what was required for the WEPP model. As a result, a simplified method was chosen to route rainfall excess on multiple OFEs in order to take advantage of the simplified routing procedures outlined in sections 4.4.1 and 4.4.2.

4.5.5 Equivalent plane

The equilibrium storage concept developed by Wu et al. (1978) is used to transform multiple OFEs into a single uniform OFE. The method is based on the relationship between the kinematic depth-discharge coefficient and the storage of water on the flow surface at kinematic equilibrium assuming constant rainfall excess. Simply put, the method computes an aggregate kinematic depth-discharge coefficient which will give the same equilibrium discharge rate at the outlet of a single OFE that disaggregated depth-discharge coefficients give at the bottom-most OFE of a series of OFEs. The relationship between storage and α is derived by integrating the equilibrium depth, h_e (m), with respect to distance to obtain the water stored on the system of OFEs at equilibrium. For constant rainfall excess, the equilibrium depth is

$$h_e = \left(\frac{L\ \nu_c}{\alpha} \right)^{1/m} \qquad [4.5.11]$$

Integrating Eq. 4.5.11 with respect to distance yield

$$\frac{1}{L} \int_0^L h_e(w)\ dw = S_e = \frac{m}{m+1}\ \frac{1}{L}\ \left(\frac{\nu_c}{\alpha} \right)^{1/m} L^{\frac{m+1}{m}} \qquad [4.5.12]$$

where S_e = the storage of water on the flow surface at equilibrium (m). The depth-discharge coefficient, α, is then

$$\alpha = \nu_c\ L \left(\frac{m}{m+1}\ \frac{1}{S_e} \right)^m \qquad [4.5.13]$$

For the case of multiple OFEs, Eq. 4.5.12 becomes

$$S_e = \frac{m}{m+1}\ \frac{1}{L_e} \sum_{j=p}^{n} \left(\frac{\nu_j}{\alpha_j} \right)^{1/m} \left[L_{j+1}^{\frac{m+1}{m}} - L_j^{\frac{m+1}{m}} \right] \qquad [4.5.14]$$

where S_e = the total storage of $n-p$ OFEs, L_e = the length from OFE_p to OFE_n, L_{j+1} = the distance from the top of OFE n to the top of OFE $j+1$, and L_j = the distance from the top of OFE p to the top of OFE j. Assuming that the sum of rainfall excess is equal to the average rainfall excess over the system of OFEs, the equivalent depth-discharge coefficient, α_e, is computed by substituting Eq. 4.5.13 into Eq. 4.5.14

$$\alpha_e = \frac{L_e^{m+1}}{\displaystyle\sum_{j=p}^{n} \frac{\left[L_{j+1}^{\frac{m+1}{m}} - L_j^{\frac{m+1}{m}} \right]}{\alpha_j^{1/m}}} \qquad [4.5.15]$$

4.6 Limitations

In general, the surface hydrology component of WEPP should perform the best for large events on a single OFE and the worst for medium events on multiple OFEs assuming that the parameter values are correct. Among specific limitations are:

Hydrologic regime

— WEPP considers only Hortonian flow or flow which occurs when the rainfall rate exceeds the infiltration rate. It does not explicitly consider variable partial area response or return flow.

Infiltration

— As with most infiltration models, the implementation of the GAML model in WEPP describes the movement of water within the soil profile at a point.

— Changes in surface infiltrability are lumped in the updated value of the effective saturated hydraulic conductivity parameter. All the infiltration parameters do not vary within an OFE nor within a single rainfall event.

— If there is a hiatus within a single rainfall event, the soil moisture is not redistributed so that the infiltration rate when rainfall restarts is the same as when the rainfall ended.

Depression storage

— Depression storage is assumed to be satisfied before runoff begins.

Rainfall excess

— Rainfall excess is computed before it is routed on the flow surface. The result is that the routed rainfall excess volume (runoff) is equal to the rainfall excess volume before routing. In nature the routed volume is less because of infiltration during the recession period of the hydrograph. For partial equilibrium hydrographs, the routed volume is reduced to account for the recession infiltration (section 4.4.3.). However, for all other cases it is not.

Peak discharge

— Flow conditions are restricted to an initial condition of no flow for all cases and an upper boundary condition of zero flow for a single OFE or the top most OFE of a cascade of OFEs.
— The approximate method to compute peak discharge will show more error when the actual hydrograph has multiple peaks than when it has only a single peak.

Multiple OFE infiltration

— Infiltration on multiple OFEs is computed using the OFE length weighted average GAML parameters of the OFEs under consideration.

4.7 References

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4.8 List of Symbols

Symbol	Definition	Units
C	Chezy coefficient	$m^{0.5} \cdot s^{-1}$
C_p	indicator if ponding ceases within an interval	m
C_u	indicator if ponding occurs within an interval	m
$c(t,x)$	the characteristic defined as a locus of points in the time space plane on which the characteristic equations have one solution	$m \cdot s^{-1}$
D_e	effective runoff duration = Q_v/q_p	s
D_r	duration of rainfall	s
D_v	duration of rainfall excess	s
F_i	cumulative infiltration depth at time i	m
F_h	unit width volume rainfall and runoff on a case three or four OFE	$m^3 \cdot m^{-1}$
F_p	unit width volume potential infiltration capacity on a case three or four OFE	$m^3 \cdot m^{-1}$
f_i	infiltration rate at time i	$m \cdot s^{-1}$
f_f	final infiltration rate when rainfall excess goes to zero	$m \cdot s^{-1}$
f_*	dimensionless infiltration rate = f_f/V_a	NOD
h	overland flow depth	m
h_e	equilibrium flow depth	m
h_p	peak overland flow depth	m
h_r	overland flow depth after rainfall excess ceases	m
h_z	overland flow depth at time t_z	m
i	index for time	NOD
j	index for distance	NOD
K_a	average effective saturated hydraulic conductivity for multiple OFEs	$m \cdot s^{-1}$
K_e	effective saturated hydraulic conductivity	$m \cdot s^{-1}$
$K_{\min}$	minimum saturated hydraulic conductivity of the soil profile	$m \cdot s^{-1}$
L	length of an OFE	m
m	kinematic wave depth-discharge exponent	NOD
Q_v	runoff volume (depth) adjusted for recession infiltration	m
Q_*	dimensionless runoff volume = Q_v/V_t	NOD
q	flow discharge per unit width of the flow surface	$m^3 \cdot m^{-1} \cdot s^{-1}$
q_p	peak discharge rate	$m \cdot s^{-1}$
q_*	dimensionless peak rate = q_p/v_c or q_p/v_a	NOD
R_i	cumulative rainfall depth at time i	m
R_{tp}	cumulative rainfall depth at the time to ponding	m
r_i	rainfall rate at time i	$m \cdot s^{-1}$
r_r	random roughness	m
S_a	average maximum depression storage for multiple OFEs	m
S_d	maximum depression storage	m
S_e	total water storage at equilibrium for multiple OFEs	m
S_o	slope of overland flow surface	$m \cdot m^{-1}$
S_p	potential upper limit of soil water storage in the upper two soil layers	m
ST	current soil moisture in a soil layer	m
t	time	s

t_a	time to kinematic equilibrium computed using average rainfall excess rate	s
t_e	time to kinematic equilibrium computed using constant rainfall excess rate	s
t_p	time to ponding	s
t_z	time characteristic from x=0, t=0 reaches the end of the plane	s
t_0	time the characteristic from the top of the plane originates	s
t_{f*}	dimensionless infiltration time	NOD
t_*	dimensionless runoff discharge time = t_e/D_v	NOD
t_{**}	time when approximate peak solution switches from Eq. 4.4.23 and 4.4.24	NOD
UL	upper limit of soil moisture storage in a soil layer	m
V_i	cumulative rainfall excess depth at time i	m
V_t	total rainfall excess depth	m
v_i	rainfall excess rate at time i	$m \cdot s^{-1}$
v_a	average rainfall excess rate	$m \cdot s^{-1}$
v_c	constant rainfall excess rate	$m \cdot s^{-1}$
v_p	peak rainfall excess rate	$m \cdot s^{-1}$
v_*	dimensionless rainfall excess rate = v_a/v_p	NOD
x	distance from top of the flow surface	m
x_z	distance where the overland flow depth equals h_z at time t_z	m
w	dummy variable of integration	-
α	kinematic wave depth-discharge coefficient	$m^{0.5} \cdot s^{-1}$
α_e	equivalent kinematic wave depth-discharge coefficient for multiple OFEs	$m^{0.5} \cdot s^{-1}$
η_e	effective porosity	$m \cdot m^{-1}$
θ_d	soil moisture deficit	$m \cdot m^{-1}$
θ_v	initial volumetric water content	$m \cdot m^{-1}$
Ψ	average capillary potential across the wetting front	m
$(\theta_d \Psi)_a$	average matric potential for multiple OFEs	m

Note: NOD stands for nondimensional variable