Research Report No. 379

A LABORATORY FLUME STUDY OF SAND BED DEGRADATION

by

Joe C. Willis

U.S.D.A. Sedimentation Laboratory
Southern Branch
Soil and Water Conservation Research Division
U. S. Agricultural Research Service
United States Department of Agriculture

October, 1965

A LABORATORY FLUME STUDY OF SAND BED DEGRADATION

by

Joe C. Willis

ABSTRACT

River reaches below sediment impounding structures are characterized by perimeter scour. Simulating such conditions of interrupted sediment inflow as experienced by these reaches, a laboratory flume study of sand bed scour or degradation was conducted. The tests were designed to describe the transition of the flume channel from a relatively high flow and transport regime to the point of incipient motion of the bed material.

The results of the flume tests are presented as time transition relationships of the mean reach bed elevation and scour rate. An equation of the form, $\eta - \eta_o = K (T - T_o)^{-m}$, was chosen to present the transition description. The parameters of this equation were determined for each test and are included as functions of unit width flow discharge. Derivations based on this relationship are included to indicate possible applications to other channels; however, only the data from the flume tests are available to support this hypothesis.

The various bed and flow regime sequences encountered during the degradation processes are described and variations for different bed materials noted. For each of the three bed materials investigated, plots of scour rate against the hydraulic factors considered to influence the transport quantity under equilibrum conditions are also included along with the corresponding critical values associated with the point at which sediment motion stops.

A LABORATORY FLUME STUDY OF SAND BED DEGRADATION

by

Joe C. Willis

Introduction:

A natural stream in its undisturbed condition is in equilibrium or at least fluctuating about long term average flow characteristics. Its gradient is adjusted so that the flow characteristics produce sediment transport capacities that balance the sediment supply from the watershed.

Should a change be induced in the factors that established the equilibrium, a new equilibrium would be in order for the channel. With channel width considerations neglected, the channel would approach its new state by either aggradation, deposit of sediment, or degradation, removal of sediment from its bed.

One of the most pronounced effects of stream changes is observed in connection with sediment impounding structures, those that impound the heavy sediment particles of which the channel bed is composed. Such structures allow almost zero sediment inflow to downstream reaches with original regimes adjusted to a certain transport capacity. Degradation results. This paper is based on a laboratory flume study of such processes.

Experimental Apparatus:

A laboratory flume was designed and constructed at the U. S. D. A. Sedimentation Laboratory to permit investigation of degradation processes under controlled, but limited, conditions. The flume channel was 16 feet long and 6 inches wide with a usable depth of 1.5 feet. One side was constructed of plexiglas etched with a grid to assist in quick reference and photographic interpretation. A rock-filled inlet occupied about 1.5 feet of the channel length and served as an energy dissipator.

An adjustable tailgate weir was provided at the downstream end of the flume. A 1/8" slot under the tailgate removed the bedload along with part of the flow. The remainder of the discharge flowed over the weir. This tailgate design allowed the lower end of the sand bed to assume the submerged angle of repose of the bed material and provided a relatively constant downstream control.

A pump was provided with a bypass and supply line arrangement permitting discharge control between 0 and 0.18 cfs; however, the usable range for this investigation was about 0.04 to 0.15 cfs. A vibratory sand feeder simulated sediment inflow from upstream reaches. The flow system was semi-recirculating, since the bed material was trapped in a settling tank and virtually sediment-free water was returned by the pump.

Instrumentation for this investigation consisted of a point gage, stream monitor adapted for low depth application, and an elbow type flow meter. The flow meter was a standard 1½" pipe elbow with 45° taps. An air-water manometer was used to measure the pressure differential across the elbow. The meter was calibrated volumetrically in place.

Experimental Procedures:

The experiments were divided into two distinct phases. These were the transition or degradation phase and the final conditions phase. The degradation tests dealt with the time sequence of degradation processes while the final conditions tests were designed to establish the conditions that would exist after all degradation had occurred.

At the start of a typical degradation test, the desired flow discharge was established. Then sediment was fed into the flume at the upper end. The initial feed was by hand until the bed had aggraded to a relatively steep slope. The vibratory sand feeder was started and the system allowed to approach equilibrium. The time required for the flow to stabilize depended on how near the feed rate was to the flume transport capacity at the time the sand feeder was started. Exact equilibrium was not required and usually only 5 to 10 minutes of sand feeder operation was necessary to establish a satisfactory flow condition. Figure 1 shows the degradation flume with the initial sand feed.

Degradation was initiated by stopping the sand feed. Measurements of bed and water surface profiles were taken as scour occurred. The initial flow discharge was maintained throughout the degradation test. Temperature control was accomplished by a flow interchange of cold water from the water supply system.

During the initial phases of the tests and during other periods when the bed was relatively plane, water surface and bed profiles were determined by point gage readings. During the initial part of the tests when the profiles were changing rapidly, point elevation readings were taken at two-foot intervals over the test reach from station 2.5' to 14.4'. About 2.5 minutes were required to complete a set of these water surface and bed readings. After about 20 minutes when the scour rate was sufficiently low, the point elevation readings were taken at one-foot intervals. About 4 minutes were required to obtain a set of these readings.

Since only the time distribution of the channel profiles was important, time = 0 was arbitrarily taken at the time the sand feed was stopped. The reference time for each set of profiles was taken at the start of each set of profile readings. A typical set of three water surface and bed profiles is shown in figures 2 and 3 respectively.

In tests when dunes developed (fig. 4), point elevations would not adequately describe the bed profiles. For such conditions the water surface profiles were still determined with the point gage, but bed profiles were recorded with the stream monitor. Two traverses with the monitor along lines about 2 inches from each wall were made to establish the bed profile. The reference time for such readings was taken as average of the time the water surface readings were begun and the time the bed traverse was ended. About 7 minutes were required to obtain a set of profiles by the stream monitor method.

The final conditions tests were designed to obtain a reasonable estimate of the point at which degradation would stop. For no sediment inflow into the test flume, the final condition is the point at which sediment motion stops. Theroretically, this point will not be reached through degradation processes for the scour rate diminishes but never quite reaches zero. However, the channel conditions approach a certain limit or asymptote.

Although the final condition for a channel cannot be reached with fixed controls, altering the control may permit extrapolation to this condition. Consider the definition sketch of figure 5. Let us assume that the flow in the test channel is as depicted by the bed and the water surface "A". With the discharge held constant, the tailgate is raised until sediment motion stops. This causes the water surface to be raised to "B". If the bed were in such a configuration that it might assume at the final state except that it is at a higher elevation and experiencing a low transport rate, the distance that the water surface is raised should equal the average distance that the bed would be lowered in reaching that point with a fixed tailgate control.

After degradation had proceeded for about 8 hours, the bed slope for each test became very small (almost zero) and remained effectively zero throughout even the longest flow period investigated (290 hours). Thus the bed slope of the final conditions test should be representative of the final bed slope or at least should be insignificantly different based on the short flume length.

After a dune pattern formed, no change was observed in the effective pattern as degradation proceeded. Even after a period of very low transport, the bed forms established by all discharges within the range employed in this investigation were effectively the same.

Thus the procedure of the final conditions tests was to allow a bed form to develop for a bed elevation somewhat lower than that obtained during the degradation tests, the base tailgate setting, and a discharge yielding a very low transport rate. The flume was allowed to run under these conditions for at least 24 hours. The elevated water surface profiles for zero transport were then obtained for about 10 discharges within a band approximately 1/3 the usable flume discharge range about the discharge that established the bed form. The tailgate was then returned to its base position and each discharge repeated to obtain the water surface profile "A" (see fig. 5).

Analysis Considerations:

A non-changing profile or gradient for a particular channel reach cannot be expected. Hydrograph flows, dunes, meanders, and other factors must impose irregularities along the channel. Even for the controlled conditions of the degradation flume, little order could be observed for the degradation processes at a single point. Initially the upstream end of the test flume would experience very high scour rates while, in certain instances, aggradation would occur near the downstream end. A short time later a very low scour rate for the upstream end accompanied by a relatively high scour rate near the downstream end might characterize the degradation phenomena.

Even though local irregularities are present, certain average values may be associated with a channel reach. Mean bed and water surface slopes may be defined as the slope of the line about which the sum of the elevation deviations over the reach is zero. Mean water surface and bed elevations may be defined as the elevations of their respective line at the mid-point of the reach.

Discontinuties imposed on channels by base levels such as reservoirs and other points of control such as points of fixed channel geometry necessitate that each channel segment between such controls possess its own particular regime. The downstream point of control becomes the pivot point for regime or gradient changes for the channel reach.

With constant sediment and fluid properties, the interaction of discharge and transport characteristics should determine the stream gradients and, for a reach at a specified distance upstream from a downstream point of control, the average bed elevation. Thus for a specified channel reach, a specified flow and a specified control point, the average bed elevation should be determined by the sediment supply or transport rate. Conversely the regime transport rate defines the average elevation of the reach subject to the above conditions. Thus:

$$G = f(\eta) \tag{1}$$

where G is the transport rate and η is the average bed elevation above some datum.

Considering the test flume with no sediment inflow, G becomes the transport rate out of the flume and the average scour or degradation rate can be written as:

$$\frac{d\eta}{dT} = \frac{G}{\gamma_s bL} \tag{2}$$

where $d\eta/dT$ is the scour rate in ft/hr, G is in lb/hr, γ_s is the bulk density of the bed material in lb/ft^3, b is the flume width in ft., and L is the length of the flume bed in ft.

Thus, from equations (1) and (2) for the test flume with constant flow discharge, the degradation rate should be defined by the average elevation of the sand bed.

$$\frac{d\eta}{dT} = F\ (\eta) \tag{3}$$

Bed Materials Used:

The scope of this investigation was limited to three different bed materials. Figure 6 gives the grain size distribution for each of the three materials.

Sand #1 was material obtained from a sand pit near Oxford, Mississippi. It was somewhat angular and had a mean particle diameter of 0.61 mm. Sand #2 was the finest of the three sands (Dm = 0.31mm) and it too was obtained from a natural deposit near Oxford. Its grains were also quite angular. Sand #3 (Dm = 0.40 mm) was obtained from the bed of Cuffawa Creek, Mississippi, and its particles were characterized by rounded corners.

Bed Regimes:

As degradation proceeded for each of the degradation tests, a transition through various bed forms or regimes was observed. The sequence of appearance of the regimes was the same for all tests with a particular bed material but variations were evident for different materials. The regimes were characterized as follows:

1. Standing waves - The first few minutes of tests involving sand #2 and #3 were characterized by standing waves and breaking waves. The wave length averaged about 0.7 ft. These waves were accompanied by sand waves of like form.

2. Flat bed - Beginning at the upstream end of the test reach, a relatively smooth bed and water surface developed about three minutes after the sand feed was stopped. A well defined separation line was evident between the flat bed and the standing waves. This line moved downstream until the entire bed was relatively flat. A similar flat bed phase was the initial regime of the tests with sand #1.

3. Standing waves or "Rooster Tails" - The flat bed phase lasted about 10 minutes and was followed by standing waves and sand waves having a wave length of about 0.3 to 0.4 ft. They would start as low, sine-type waves and develop to the breaking point. They were characterized by one or two peaks resembling "rooster tails". This type flow lasted from only 2 to 3 minutes for sand #1 to about 10-15 minutes for sand #3

4. Final regimes - Near the end of the "rooster tail" phase, low dunes developed under the waves and the wave-dune pattern moved slowly downstream. For sand #1, the dunes disappeared with the waves and the bed remained relatively flat throughout the remainder of the test. The dunes also disappeared for the sand #3 tests but small ripples soon appeared and continued to develop until a pronounced dune pattern was present. The low dunes of the sand #2 tests developed rapidly into a dune pattern. The dune patterns of the #2 and #3 sands

persisted to the end of the degradation tests and similar patterns developed under the conditions of the final conditions tests.

Data Analysis:

From the degradation tests profiles the following basic factors were calculated:

1 - Mean reach bed elevation, η

2 - Mean water surface elevation, $\bar{z}$

3 - Average bed slope, Sb

4 - Average water surface slope, Sw

For profiles described by point elevations, the above factors were calculated by the least squares method. For the stream monitor bed traces, the bed slope and elevation were determined as follows: (1) Mean bed elevation was determined by planimetering between the recorder trace and a reference elevation, (2) Mean bed slope was estimated from the differences in mean elevations for the upstream and downstream halves of the test reach.

From equation (3), that the transition of scour rates and mean bed elevations should define unique time functions for individual tests can be deduced. Arithmetic plots of average bed elevation against time yielded curves which were hyperbolic in shape. Thus a hyperbolic relationship of the following form was used to describe the time transition of degradation for these flume tests:

$$\eta - \eta_o = (\eta_1 - \eta_o) \left(\frac{T - T_o}{T_1 - T_o} \right)^{-m} \qquad (4)$$

where η and η_1 are bed elevations based on an arbitrary datum referenced to the base flume tailgate position and taken at times T and T_1 respectively after the sand feed was stopped. η_o and T_o are the elevation and time asymptotes of the hyperbolic relationship and m may be thought of as the degradation time exponent.

If we let $T_1 - T_o$ be equal to unity and let the corresponding value of the quantity $(\eta_1 - \eta_o)$ be equal to K, the unit time elevation, then (4) reduces to:

$$\eta - \eta_o = K(T - T_o)^{-m} \quad \text{or} \quad \eta_* = K\, T_*^{-m} \qquad (5)$$

Equation (5) may be differentiated to obtain the degradation or scour rate as follows:

$$\frac{d\eta}{dT} = -mK\,(T - T_o)^{-m-1} = -m \left(\frac{\eta - \eta_o}{T - T_o} \right) \qquad (6)$$

$$\text{or} \quad \frac{d\eta}{dT} = -m \frac{\eta_*}{T_*}$$

Degradation rate could seemingly be determined by averaging between data points as $\Delta\eta/\Delta T$; however, the errors in each data point would result in considerable error between successive points. A higher degree of significance can be assigned to the overall transition of bed elevations than to any two particular elevations. For this reason, a method of visual curve fitting employing equation (6) was chosen to establish the time sequence of scour rates.

For a given finite sequence of experimental coordinate points that describe a hyperbolic line segment, there are numerous hyperbolas having, within the experimental error of the points, corresponding segments. Each of these hyperbolas have different asymptotes and m and K parameters. Thus the goal of the visual curve fitting procedure was to determine the m and K parameters most consistent with experimental data.

The first step in the scour rate determination was to establish on tracing paper a series of arithmetic plots of relations: $\eta_1 = K' T_1^{-m'}$ for values of K' and m', yielding curves corresponding in shape for an eight hour segment to the data curves, $\eta = f(T)$. These "reference" curves were determined essentially by trial and error.

The second step was to visually superimpose the reference curves over a data curve and select the reference curve corresponding most nearly in shape to the data curve. Figure (7) is a hypothetical representation of this procedure. The lines $T = a$, $\eta = c$, $T_1 = a$, and $\eta_1 = c$ are corresponding reference lines for the data and reference curves respectively. T_s and η_s are the time and elevation axis translations between the curves; therefore, T_1 and η_1 values may be established for each data point of the data curve as follows:

$$\eta_1 = \eta - \eta_s \quad \text{and } T_1 = T - T_s \tag{7}$$

The third step was to calculate the scour rate. A preliminary estimate of the scour rate associated with each data point was calculated by equation (6) as:

$$\frac{d\eta}{dT} = m' \frac{\eta_1}{T_1} \tag{8}$$

The terms η_s and T_s are not necessarily the translations that establish the true asymptotes of the data curve. From the final conditions tests, a reasonable estimate of the elevation asymptote, η_o, can be determined as follows (refer to fig. 5).

$$\eta_o = \eta - (z - z_A) - 1\ (Sw_A - Sw_B) \tag{9}$$

Figure 8 presents the final elevation of the flume bed as a function of flow discharge for each bed material. The final elevation or asympote for each degradation test was determined from the appropriate regression curve at the flow discharge of that test.

Values of $\eta - \eta_o = \eta_*$ were then calculated. With $T - T_o = T_* = T_1 - T_o'$, equation (6) may be written in the following form:

$$\frac{d\eta}{dT} T_1 = T_o' \left[\frac{d\eta/dT}{\eta_*}\right]^{-m} \tag{10}$$

or $Y = T_o' X^{-m}$

The Y and X coordinates describe a straight line with a slope of T_o' and a Y intercept of -m. The values of scour rate used in establishing the coordinates were determined by equation (8). Use of the T_1 values in this solution was arbitrary; however, they were usually near T_* (T_o' was small). X and Y coordinates could be established for any time values following the data sequence. Values of m and T_o' were computed for each test by least squares.

To complete the description of the time transition of degradation, the value of K in equation (5) must be determined. From equation (5):

$$\log \eta_* = \log K - m \log T_* \tag{11}$$

Logarithms were determined as indicated and the value of K for each test was determined by least squares.

Figures 9 and 10 present m and K as functions of flow discharge with bed material as the parameter. If mean diameter of the material alone is considered, inconsistencies are apparent. The m curve of sand #3 (0.40 mm. mean diameter) has curvature that might be expected for a material between sands #1 (0.61 mm.) and sand #2 (0.31 mm.); however, it is displaced to lower values of m than are the curves for either of the other materials. Similar inconsistencies are apparent among the final elevation curves of figure 8 and the K curves.

Two possible explanations are offered for these apparent inconsistencies. First, the shape of the sand grains could affect the transportability of the material. The grains of sand #3 were rounded while those of sands #1 and #2 were quite angular. Second, the existence of a laminar sublayer on the backs of dunes could result in lower transport of finer material for the same flow characteristics. For resulting flows of the elevated control in the final conditions tests, dye injection revealed laminar traces extending to the dune crests of sands #2 and #3. This laminar sublayer was also evident for the relatively flat bed of the sand #1 tests. Smaller particles within a certain size range might be shielded by this sublayer while larger particles might protrude from this layer and be transported at lower bed elevations and transport intensities.

Obviously, such reasoning can apply to only a limited range of bed material sizes.

Although the values of K and m do not conform to a definite relationship based on bed material particle size, they apparently conform to a definite relationship between each other. Figure 11 presents the K vs. m relationship for each of the bed materials and flow discharges investigated.

From the values of m, η, and T as computed by the preceding procedure, new estimates of scour rate were computed by equation (6). These values differed only slightly from those determined from equation (8) and the visual curve fitting procedure. These final estimates were used in further analyses.

<u>Reach Length Modeling Hypotheses</u>:

By equation (2), the scour or degradation rate depends on not only the transport rate but also the length of the sand bed from which the sediment load was derived. The load passing the downstream point of control was removed from the bed of a reach extending to the upstream controls where the sediment inflow was zero. If the sediment inflow at the upstream control were a certain constant value other than zero, then the load contributing to degradation, G, would be the increase in the transport rate over the reach.

The time parameter T_* may be eliminated between equations (5) and (6) to obtain the following relationship:

$$\frac{d\eta}{dT} = -mK^{-1/m} \eta_*^{1+1/m} \tag{12}$$

which is the form of equation (3) as indicated by the test results. Since the product, $-mK^{-1/m}$, is numerically representative of the scour rate for a mean reach elevation above final of 1 ft., use of this equation represents an extrapolation of the test data. However, it will serve to illustrate the adjustments that a different reach length may require.

For two particular test flumes or channels with all characteristics identical except for the distance between grade controls, identical bed gradients should be established for equal sediment loads and flow discharges. Assuming that the change in bed elevations at the downstream controls will be negligible for the channels as degradation proceeds, the mean elevations of the reaches above the zero transport or degradation final elevation will be related as:

$$\eta_{*_2} = \frac{L_2}{L_1} \eta_{*_1} \quad \text{where} \tag{13}$$

L_1 and L_2 are the reach lengths.

For identical sediment loads contributing to degradation, the degradation rates will be related as:

$$\frac{d\eta}{dT_2} = \frac{L_1}{L_2}\,\frac{d\eta}{dT_1} \tag{14}$$

With the degradation equation for the channel of length L_1 between controls established as:

$$\frac{d\eta}{dT_1} = -mK^{-1/m}\,\eta_{*_1}^{1+1/m} \tag{15}$$

The relation for the second channel can be determined as follows:

Substituting (14) in (15):

$$\frac{d\eta}{dT_2} = -\frac{L_1}{L_2}\,mK^{-1/m}\,\eta_{*_1}^{1+1/m} \tag{16}$$

Solving (13) for η_{*_1} and substituting in (16):

$$\frac{d\eta}{dT_2} = -m\left[\left(\frac{L_2}{L_1}\right)^{2m+1} K\right]^{-1/m} \eta_{*_2}^{1+1/m} \tag{17}$$

Thus correcting the degradation relation for different distances between controls requires the adjustment of only the unit time elevation factor K so that

$$K_2 = \frac{L_2}{L_1}^{2m + 1} K_1 \tag{18}$$

The preceding channel length relations are presented as a possible modeling technique between similar conditions of channel geometry, flow, and sediment properties. Additional data like that presented in this report would be required, but a knowledge of flow discharge and sediment properties should yield appropriate values of m and K for a reference control spacing. The bed material transport rate might be measured or calculated by reliable methods. Equation (2) could then be used to calculate the average scour rate considering that the entire transport contributed to degradation. Next, the value of η_{*_2} could be calculated from equation (17).

This value of η_{*_2} is, in effect, the prediction of the maximum extent of degradation for it represents the average distance that the bed of the channel would be lowered in reaching the zero transport elevation. When a reduced transport regime is considered (i.e., with a sediment inflow of Gi), the elevation off final for zero transport at a transport of Gi could be computed by equation (17) with $\frac{d\eta}{dT_2} = \frac{Gi}{\gamma bL}$. With η_{*_2}' denoting this elevation off final, the extent of degradation would be given $\eta_{*_2} - \eta_{*_2}'$.

Channel Hydraulic Factors Affecting Degradation:

The hydraulic factors which influence degradation should vary little from those that determine the sediment transport quantity under equilibrum conditions. The average scour rate for a channel reach at any time must depend upon the capacity for the flow at that time to transport the bed material from the reach. This process involves initiation of motion of the bed materials at various points along the reach and sustaining this motion for the remainder of the channel length.

One of the problems associated with degradation tests is that most of the hydraulic factors constantly change. As the channel bed is lowered by a constant flow discharge, energy slope decreases, depth increases, velocity decreases, etc. The influence of these changing factors on degradation rates was investigated from three concepts of sediment motion.

Velocity Concept:

The velocity concept assumes that the transport quantity can be described by a function of mean flow velocity and sediment properties. For local scour problems, Exner's first erosion equation as used by Leliavsky (1955) in connection with dunes and ripples might be employed

$$\frac{d\eta}{dT} = \epsilon \frac{d\bar{V}}{dx} , \qquad (19)$$

where $\bar{V}$ is the mean flow velocity, ϵ is an erosion coefficient and x represents distance along the channel.

Attempts to apply equation (19) to the average scour rates and velocity gradients of the degradation flume channel were unsuccessful. At the instant that degradation was initiated in the test flume, the velocity gradient along the channel was often near zero and in some cases positive $\left(\bar{V}_x > \bar{V}_{x + \Delta x}\right)$; however, the highest scour rate was associated with this point. For the degradation tests, mean velocity alone presented a more reliable description of the degradation rates.

Mean velocity was calculated for each set of water surface and bed profiles by the formula $\bar{V} = Q/bh$ where $H = Z - \eta$. Figures 12 through 14 present the $d\eta/dT$ vs. $\bar{V}$ relations for each of the bed materials investigated. The low flow depths and standing wave irregularities probably contributed to the scatter in the high scour rate ranges. In the very low scour rate ranges a poorly defined discharge dependency was observed.

If the velocity concept will apply to all transport ranges, there must be a definable velocity below which the transport capacity is insufficient to transport the material of the channel bed. This critical velocity for sediment motion should correspond with that established for the final conditions test with $h = Z_B - \eta$. These velocities were calculated and are shown in figure 15 as functions of flow discharge.

Tractive Force Concept:

In his description of degradation processes, Tinney (1962) used Duboy's equation to describe bed-load movement. By differentiating that equation with respect to distance along the channel, he obtained $d\eta/dT$ as a function of tractive force and the tractive force gradient. Since both velocity and tractive force gradients for a rectangular channel are based on the depth gradient dh/dx, neither can be applied to average values for a reach. For the degradation tests of this investigation, average tractive force values were applied.

According to the tractive force concept, the transport capacity of the flow should depend upon the forces that the flow exerts on the channel bed. The average force exerted on the channel perimeter is given as γRSe where γ is the specific weight of the water, R is the hydraulic radius and Se is the energy gradient. If the channel were infinitely wide, the average tractive force on the bed would be given by γhSe.

Correcting the values of γhSe and γRSe by popular wall correction methods was not attempted because of the extensive calculations that a changing Reynolds' number would require. A simplified correction method was devised and employed based on the following assumption: (1) The tractive force on the smooth flume wall is uniform, and (2) the shear stress distribution away from a wall or the bed is linear.

Applying a force balance for fluid elements within the flow that employs assumption (2) above defines the shear stress on the bed at the stream center line as γhSe. Since shear stress is defined as $\mu \frac{du}{dy}$ at the bed, the linear distribution of shear away from the wall near the bed would require a velocity distribution of the second degree over the bed and also a similar shear stress distribution on the bed. We shall assume that this shear stress distribution may be approximated by a parabola having the value γhSe at the stream center and the value τ_{ow} (the wall shear stress) at the wall.

The parabolic distribution with boundary values as specified yields the following expression for the average bed shear stress:

$$\bar{\tau}_b = 1/3\ (2\ \gamma hSe + \tau_{ow}) \qquad (20)$$

Equating the total shear force on the channel perimeter to the sum of the forces on the bed and wall yields a relationship for τ_{ow} in terms of γhSe and the aspect ratio.

$$\tau_{ow} = \frac{\gamma hSe}{1+6\ h/b} \qquad (21)$$

Substituting (21) in (20).

$$\bar{\tau}_b = \gamma h Se \frac{1+4\ h/b}{1+6\ h/b} \tag{22}$$

The short length of the flume test channel resulted in highly variable and appreciable differences between the average water surface and bed slopes as degradation progressed. Therefore, acceleration correction was necessary in the energy slope determination.

The equation for the specific energy head at a point is:

$$H = \frac{V^2}{2g} + z = \frac{Q^2}{b^2gh^2} + z \tag{23}$$

Equation (23) may be differentiated to obtain the energy gradient:

$$Se = -\frac{dH}{dx} = \frac{Q^2}{b^2gh^3}\frac{dh}{dx} - \frac{dz}{dx} = Sw + \frac{Q^2}{b^2gh^3}(Sb-Sw) \tag{24}$$

where Sb and Sw are the bed and water surface slopes respectively.

For each set of water surface and bed profiles, Se was calculated by equation (24) using the average profile slopes, and tractive force was calculated by equation (22). Figures 15 through 18 present the average degradation rate as a function of these tractive force values. Corresponding values were calculated for the final conditions tests and are illustrated in figure 19 as a function of flow discharge. For these values acceleration correction was neglected and Se was taken equal to Sw.

The degradation rate vs. shear stress relationship is poor especially for sands #2 and #3 for which the moving dune patterns probably contributed to the scatter in the lower ranges.

Flow discharge dependency is evident from the final conditions results. This discharge dependency could indicate that the method for calculating tractive force is not quite right; however, it could also suggest a changing scale of turbulence superimposed on the average tractive force value.

Although the scatter of the degradation rate - tractive force plots appears almost random, relatively large tractive force values were generally accompanied by relatively small mean velocity values. Such would indicate that some combination of velocity and tractive force might best describe the degradation rate.

<u>Steam Power Concept:</u>

A logical combination of velocity and tractive force is steam power. A certain amount of energy is associated with the motion of sediment particles. The energy required to overcome the frictional resistance of the bed, initiate sediment motion, and sustain this motion is acquired at the expense of the energy of the flow. Thus the rate of sediment motion should depend on the rate that the flow expends energy on or transfers energy to the bed. This rate of expenditure of energy or stream power is given by:

$$P_b = \bar{\tau}_b \bar{V} \tag{25}$$

where P_b is the average power delivered by the flow over the test reach per square ft. of bed area.

Figures 20 through 22 present plots of degradation rates against stream power for the transition or degradation tests. These relationships are better defined than those of velocity and tractive force; however, the final conditions tests still indicate a discharge dependency (fig. 23).

Conclusions:

For the limited conditions of this investigation the following findings are summarized:

1- As degradation proceeded, the scour rate diminished from that equivalent to the initial transport rate to zero asymptotically with time and average reach bed elevation. The time transition of the average reach bed elevation was described by the relation: $\eta - \eta_o = K(T - T_o)^{-m}$, where η_o, K, and m are functions of flow discharge and sediment properties.

2- Mean diameter of the bed material did not present an adequate description of bed material properties active in degradation processes.

3- Application of the time transition relation in (1) above to channel reaches with different lengths between controls should require adjustment of the unit time elevation factor K so that $K_2 = \left(\frac{L_2}{L_1}\right)^{2m + 1} K_1$.

4- The bed regimes and wave forms observed as degradation proceeded were almost identical for a particular bed material and the discharge range investigated, but the different bed materials presented different bed form sequences for the lower transport conditions.

5- Of the hydraulic factors investigated, stream power presented the best description of the degradation rate; however, a slight discharge dependency might require additional adjustment for extended discharge ranges.

REFERENCES CITED

1 Leliavsky, Serge, An Introduction to Fluvial Hydraulics, Constable & Company, Ltd., London, 1955.

2 Tinney, E. Roy, "The Process of Channel Degradation", Journal of Geophysical Research, Vol. 67 No. 4, April, 1962.

LIST OF SYMBOLS

a - time reference value for axis translation

b - channel width

c - elevation reference value for axis translation

Dm - mean diameter of the bed material

G - sediment transport rate or sediment transport rate out of the test reach

Gi - sediment inflow rate

H - specific energy head

h - mean reach flow depth

K - unit time elevation factor

K_1 - K for a reference reach length L_1

K_2 - K for a reach length of L_2

K' - visual translation estimate for K

L - length of the test reach

L_1 - length of a reach between controls for which the degradation transition relation is known

L_2 - length of a reach between controls for which the transition relation is desired

l - distance from the downstream control

m - degradation time exponent

m' - visual translation estimate of m

P_b - power per unit of bed area

Q - volumetric flow discharge

R - hydraulic radius of the flow

Sb - mean bed slope

Se - energy gradient

Sw - mean water surface slope

T - time measured from the end of the sand feed

T_o - time asymptote of the degradation transition relation

T_1 - reference time values or time values established by the visual curve translations

T_o' - time shifts from the visual translation values T_1

T_s - time shifts established by the visual curve fitting procedure

T_* - $T - T_o$

$\bar{V}$ - mean flow velocity

x - distance along or across the channel bed

X - abscissa points described by $\frac{\partial\eta/\partial T}{\eta_*}$

Y - ordinate points described by $\frac{\partial\eta}{\partial T} T_1$

$\bar{z}$ - mean water surface elevation

γ - specific weight of the water

γ_s - bulk specific weight of the bed material

ϵ - an erosion coefficient

η - average sand bed elevation based on a reference datum

η_1 - reference bed elevation or bed elevation values established by the visual curve fitting procedure

η_o - final zero transport bed elevation

η_s - elevation shifts established by the visual curve fitting procedure

η_* - bed elevation off the final zero transport elevation

$\bar{\bar{\tau}}_b$ - average shear stress on the sand bed.

τ_{ow} - shear stress on the channel side walls

LIST OF FIGURES

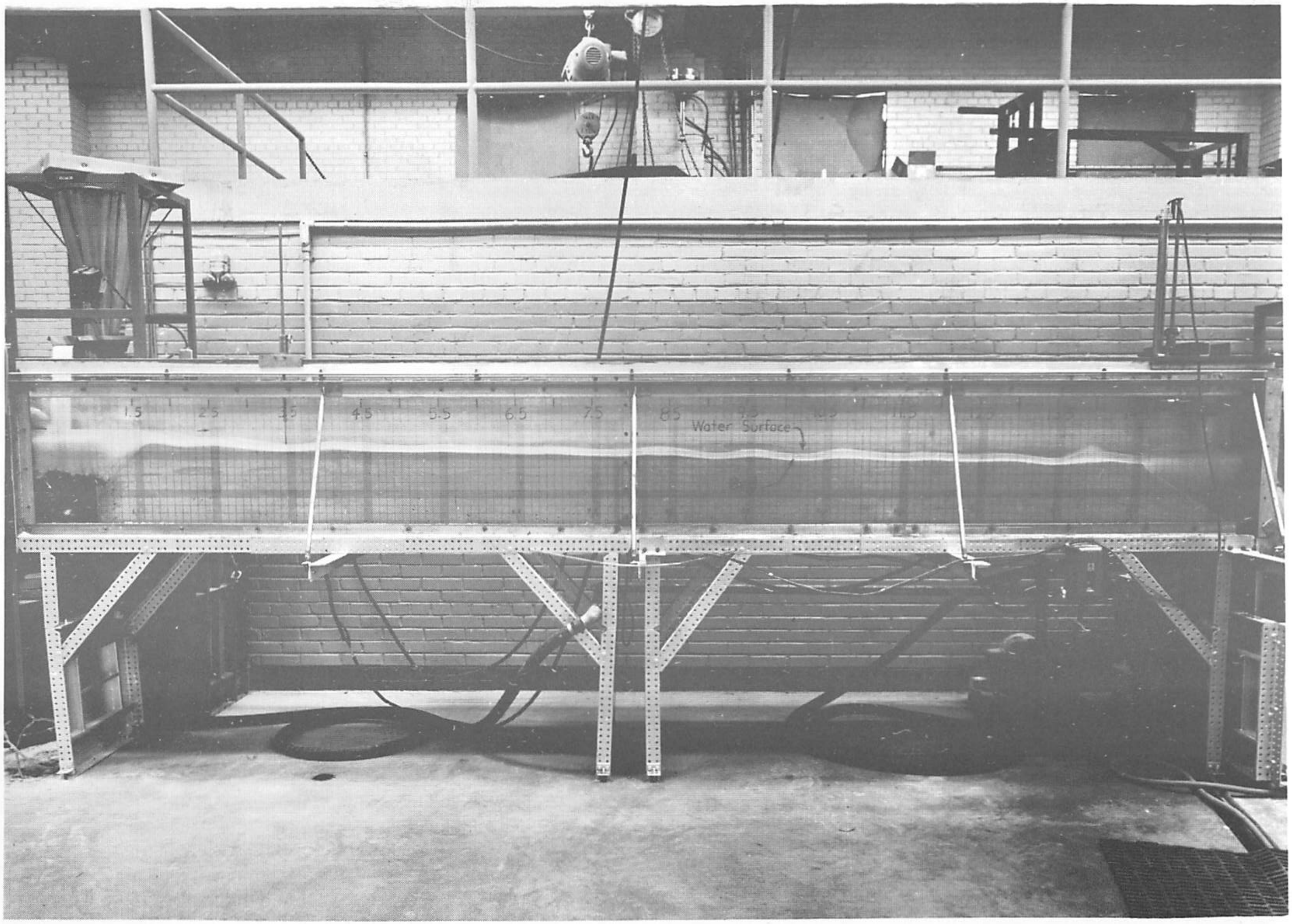

FIGURE 1. -- Degradation flume with flow near equilibrum for initial sediment feed-sand No. 2 with Q= .099 cfs

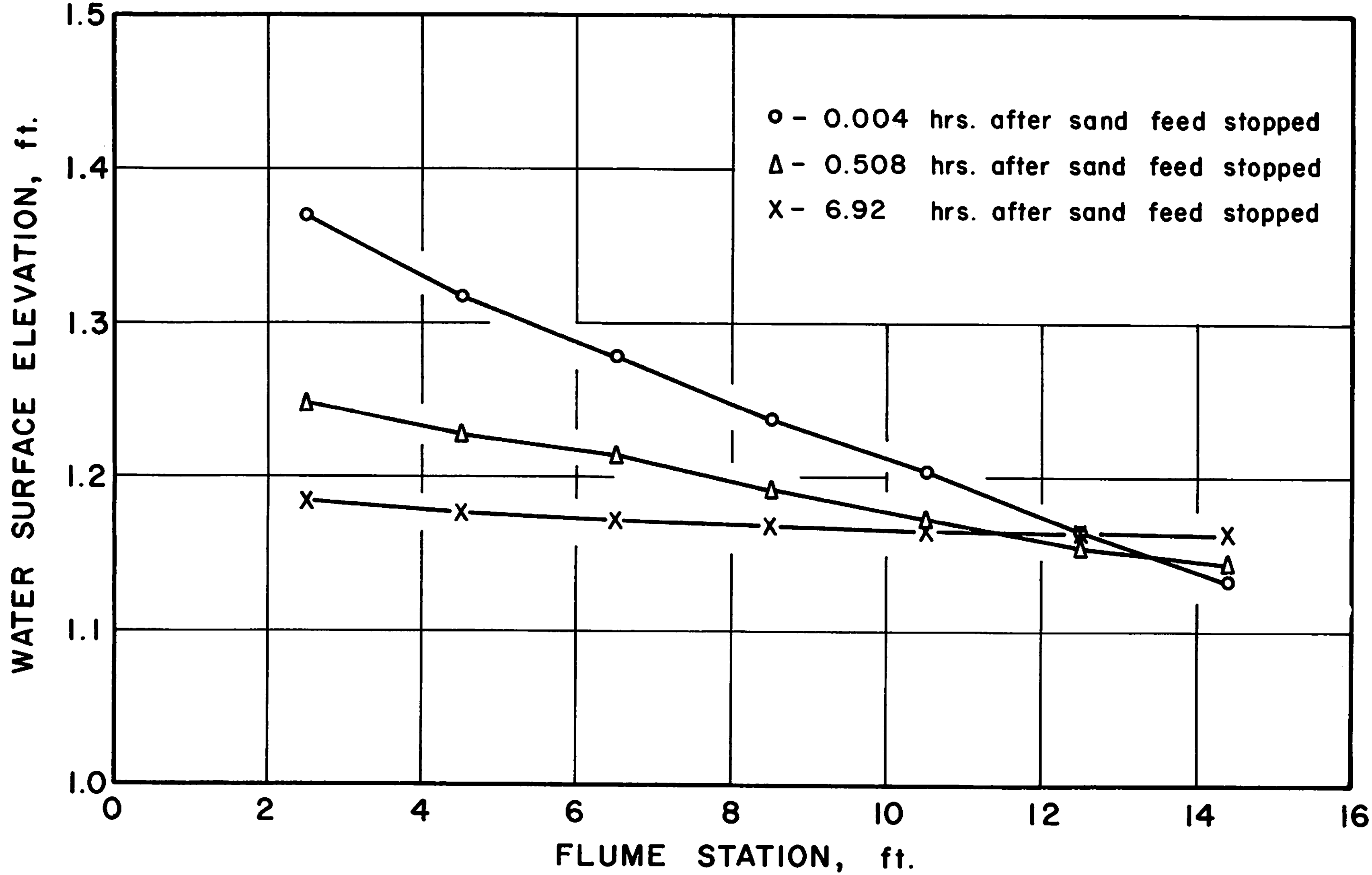

FIGURE 2.--Typical water surface profiles for sand No. 1 and Q=.063

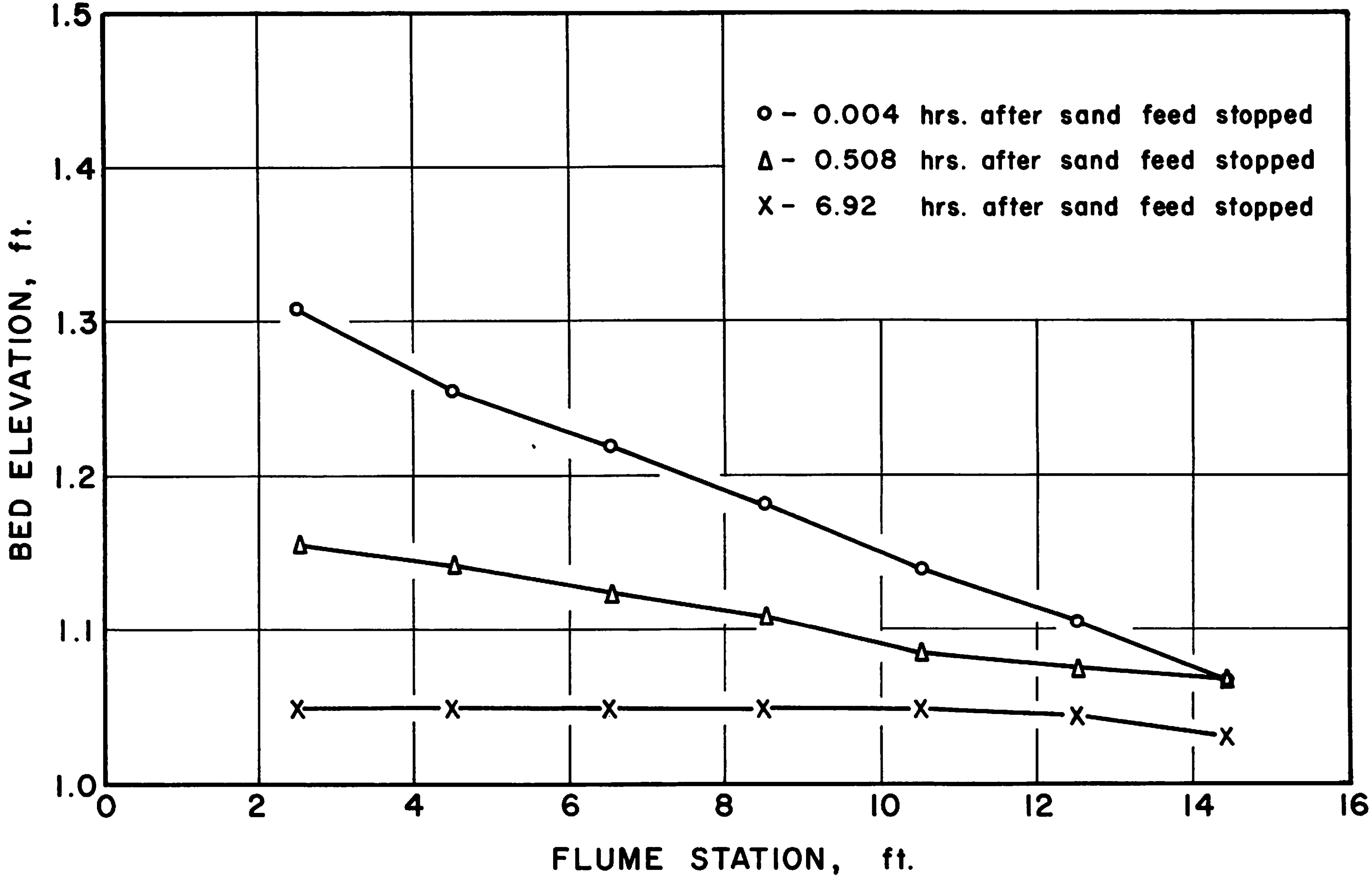

FIGURE 3.--Typical bed profiles for sand No. 1 and Q=.063

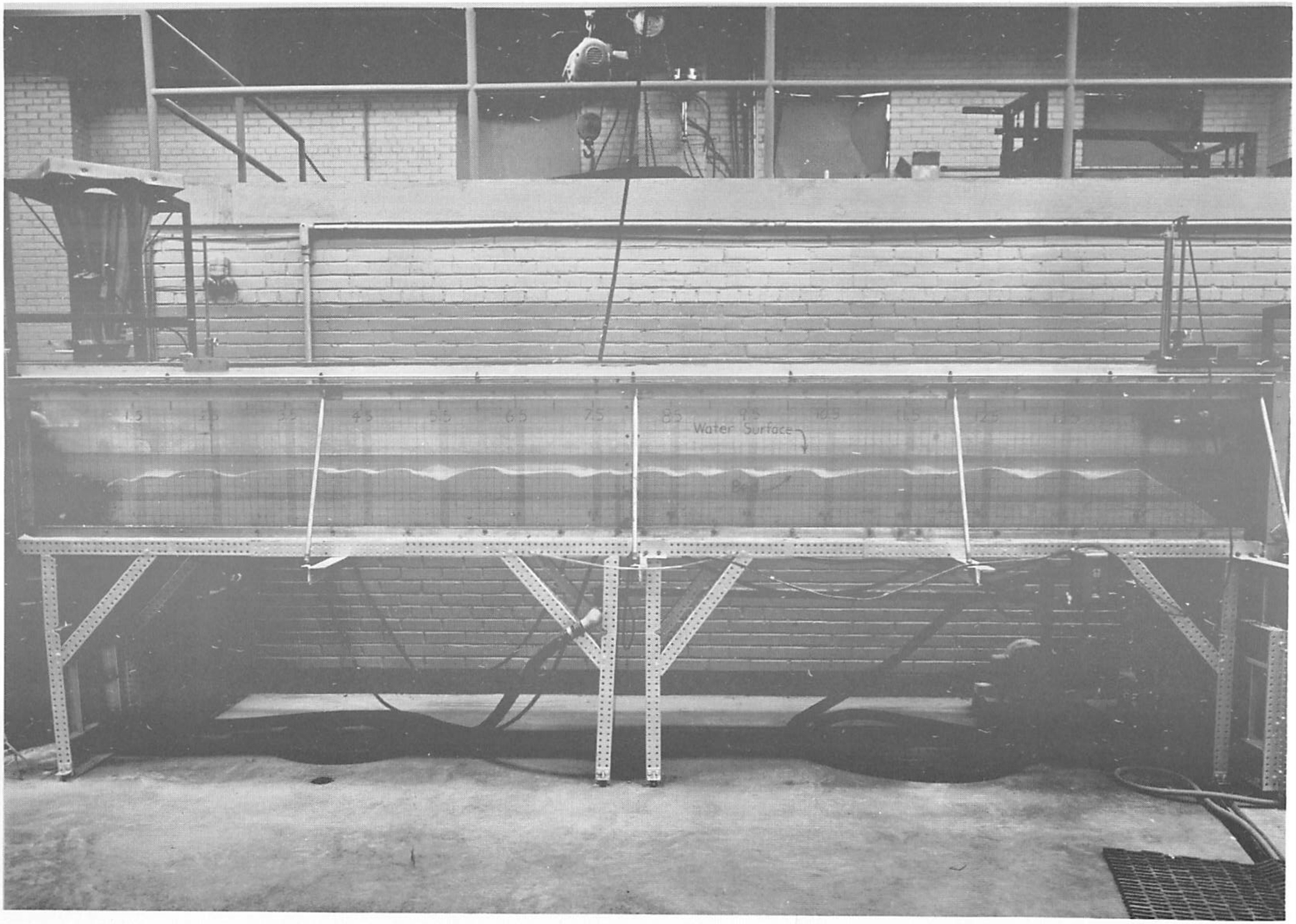

FIGURE 4.-- Degradation flume seven hours after inital sand feed was stopped - Sand No. 2 with Q = .099 cfs

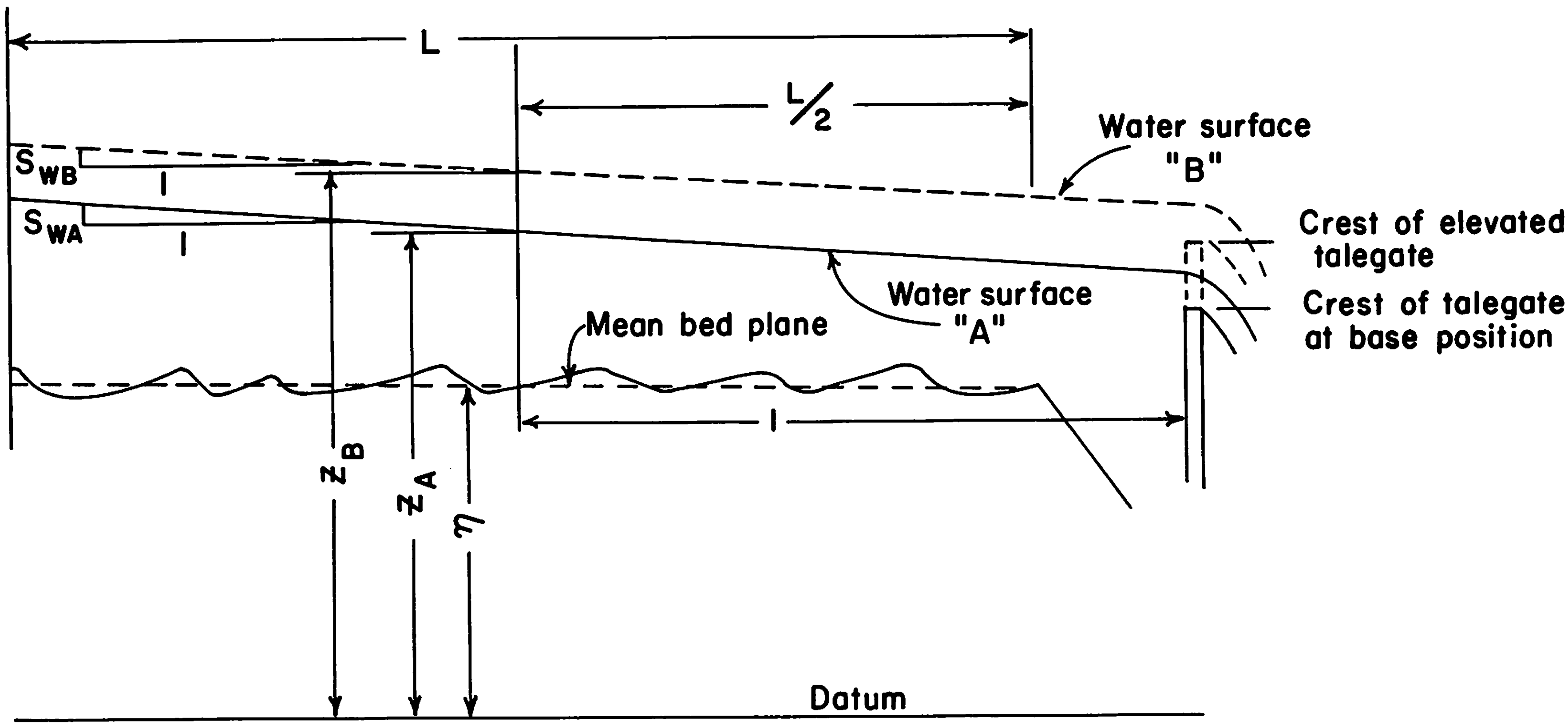

FIGURE 5.-- Definition sketch- Final conditions procedure

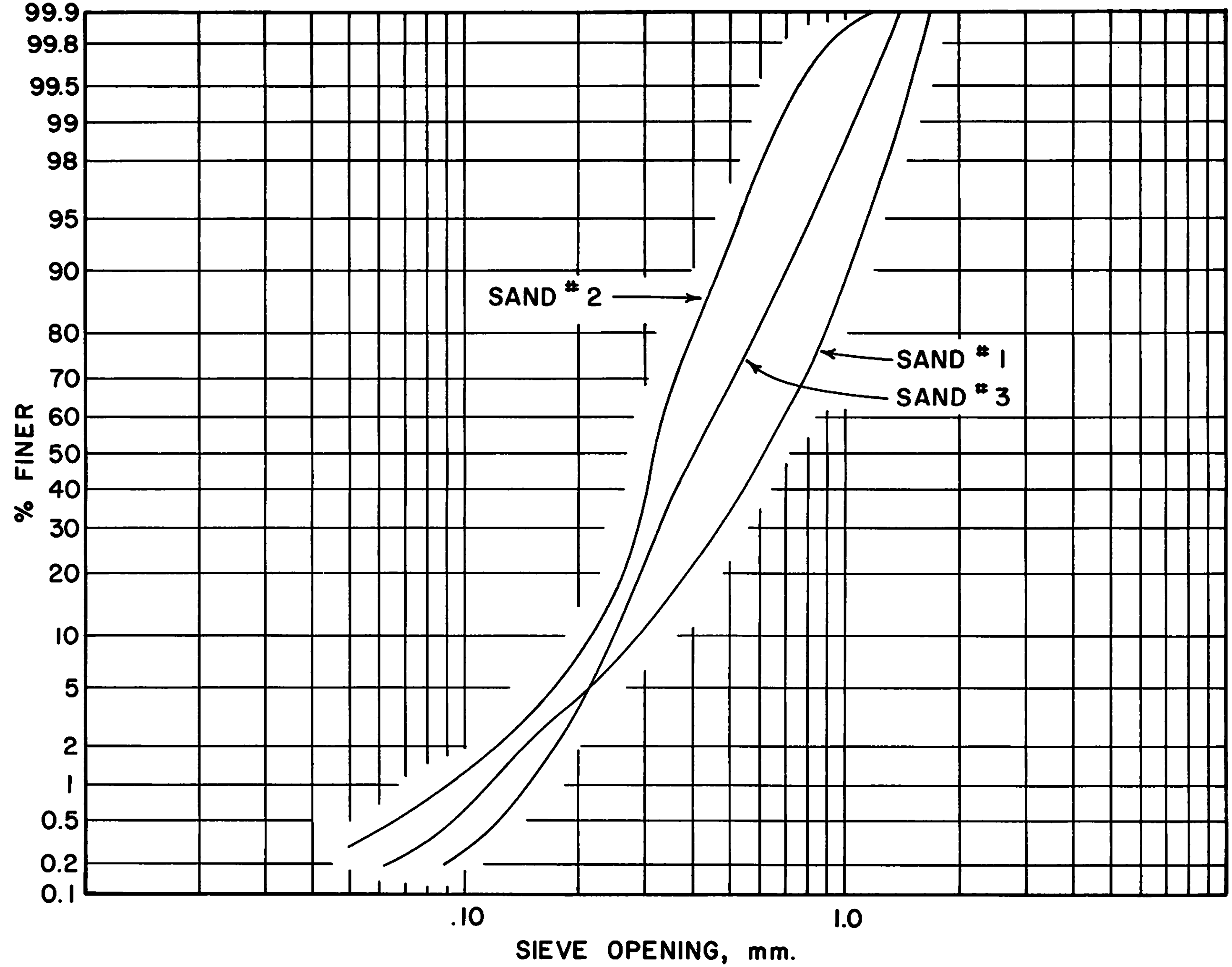

FIGURE 6.--Sieve analysis of the bed materials

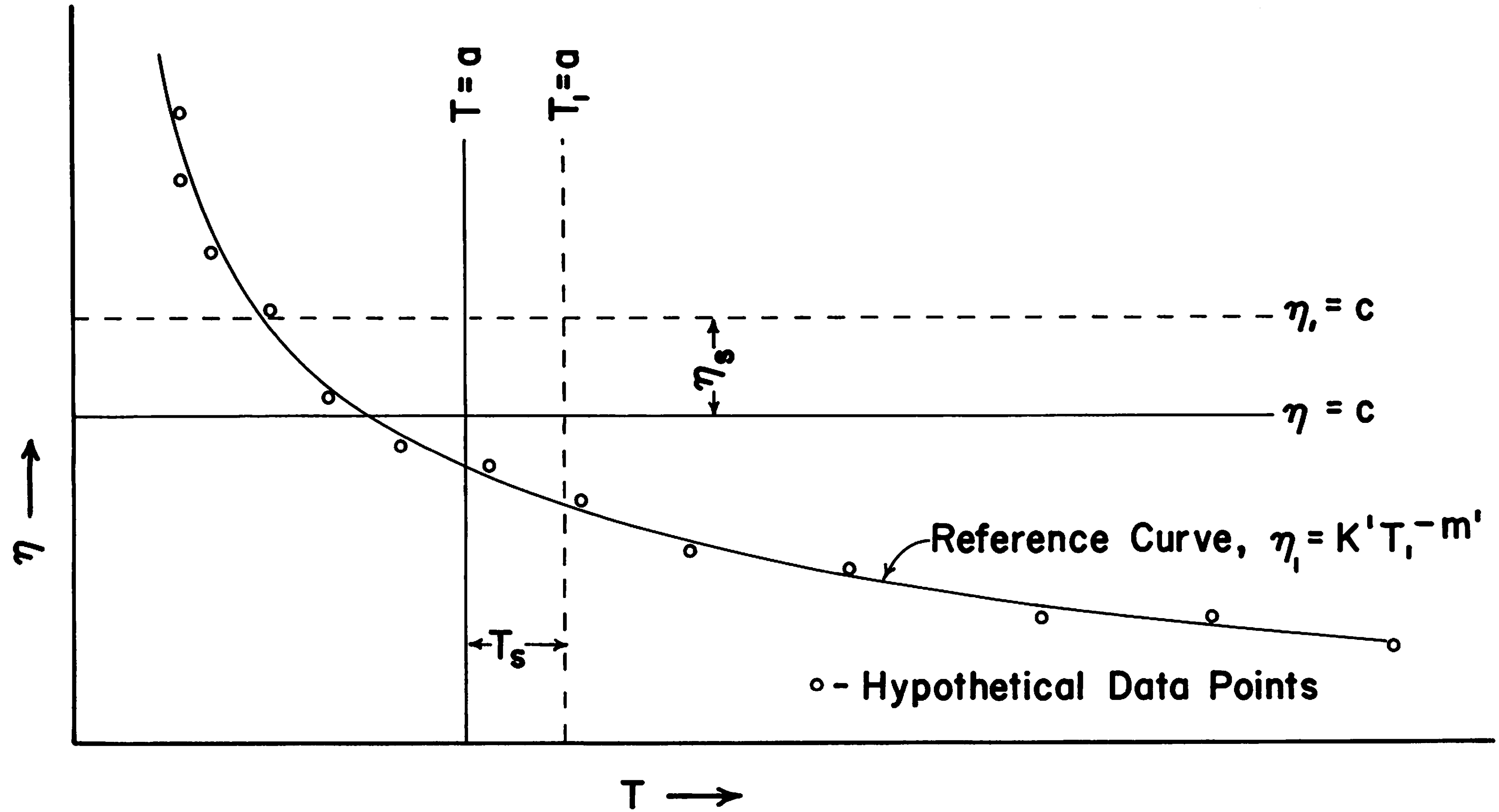

FIGURE 7.-- Definition sketch – Visual curve fitting procedure

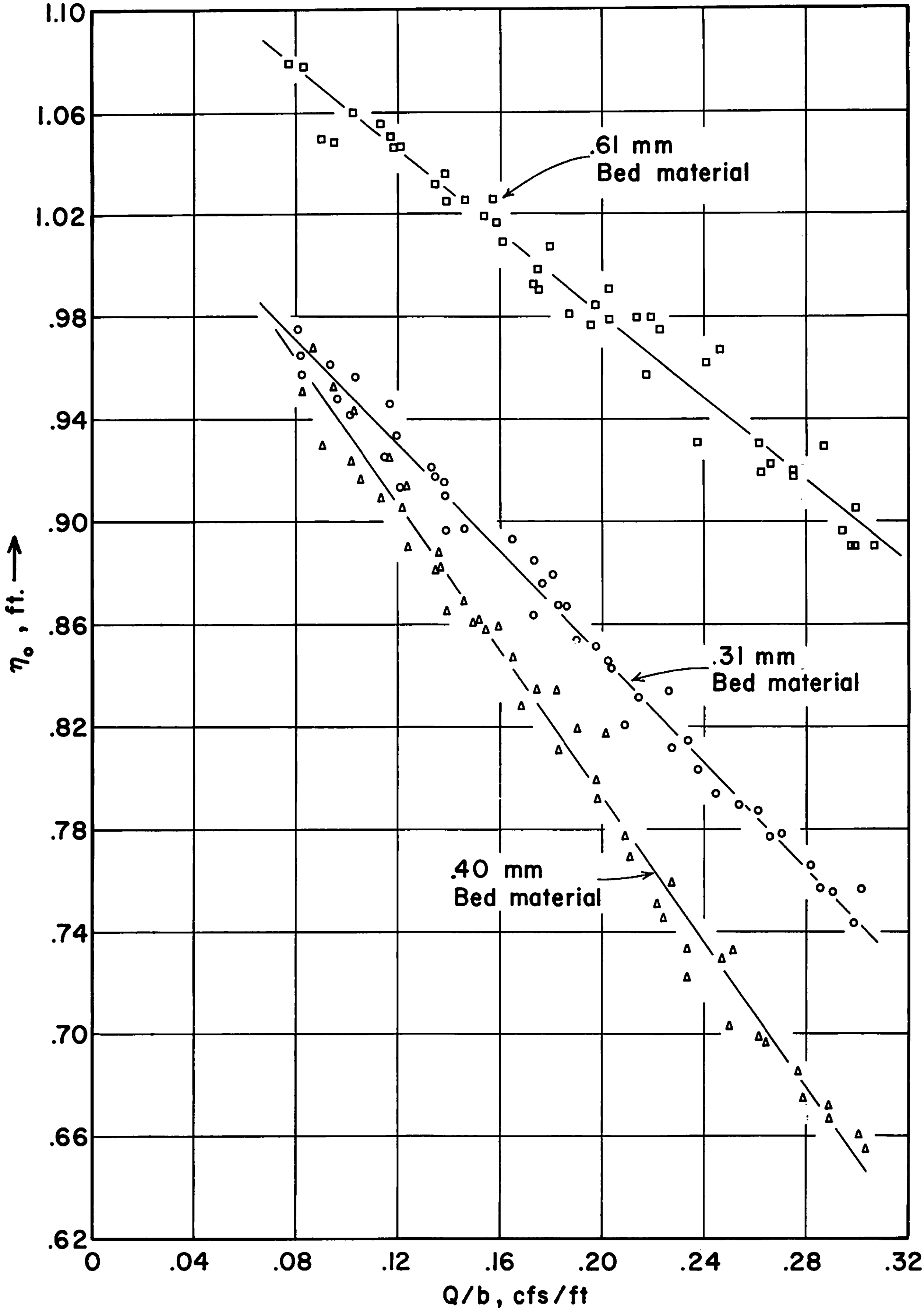

FIGURE 8.-- Limiting elevations for degrading beds

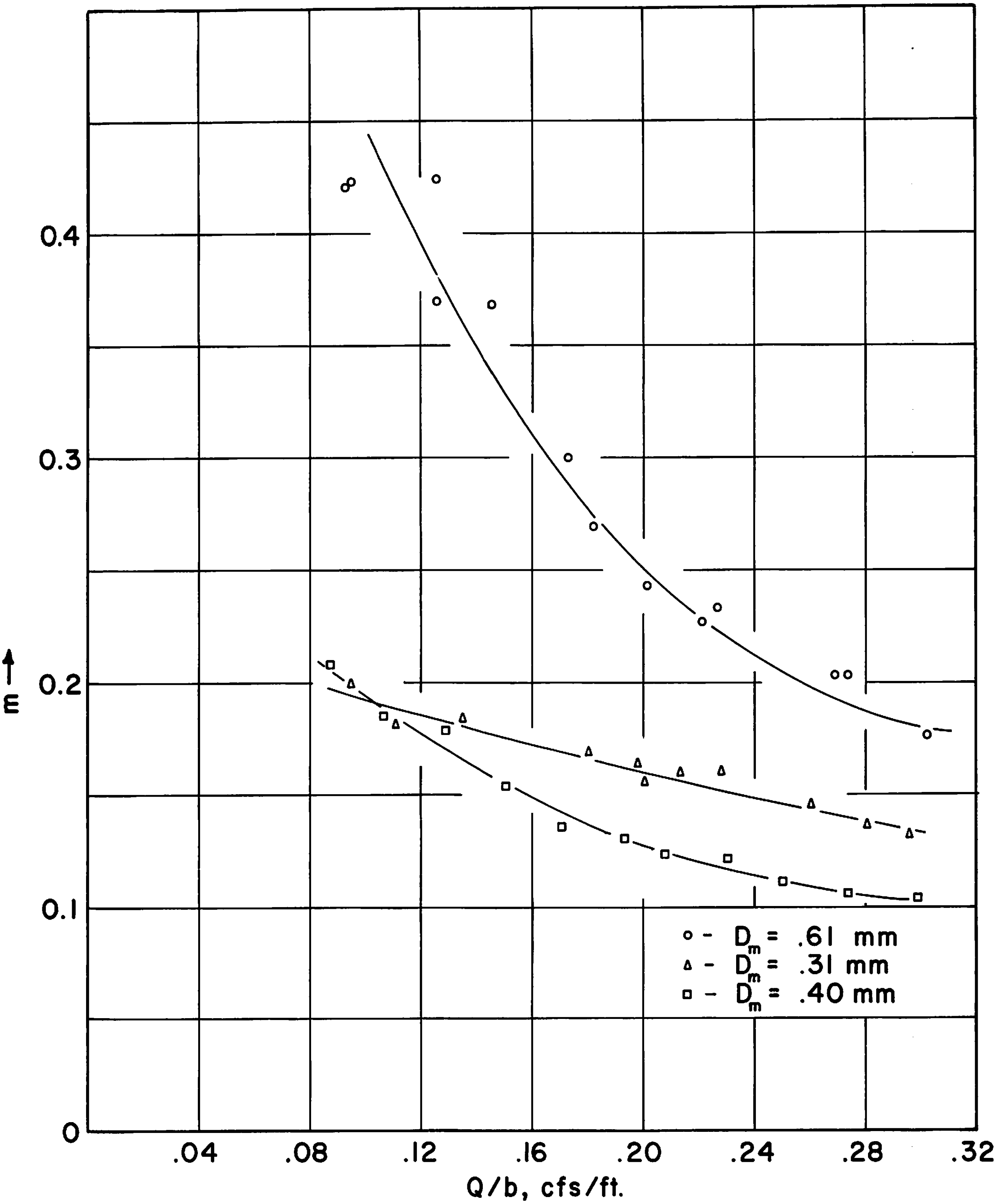

FIGURE 9.--Degradation time exponent

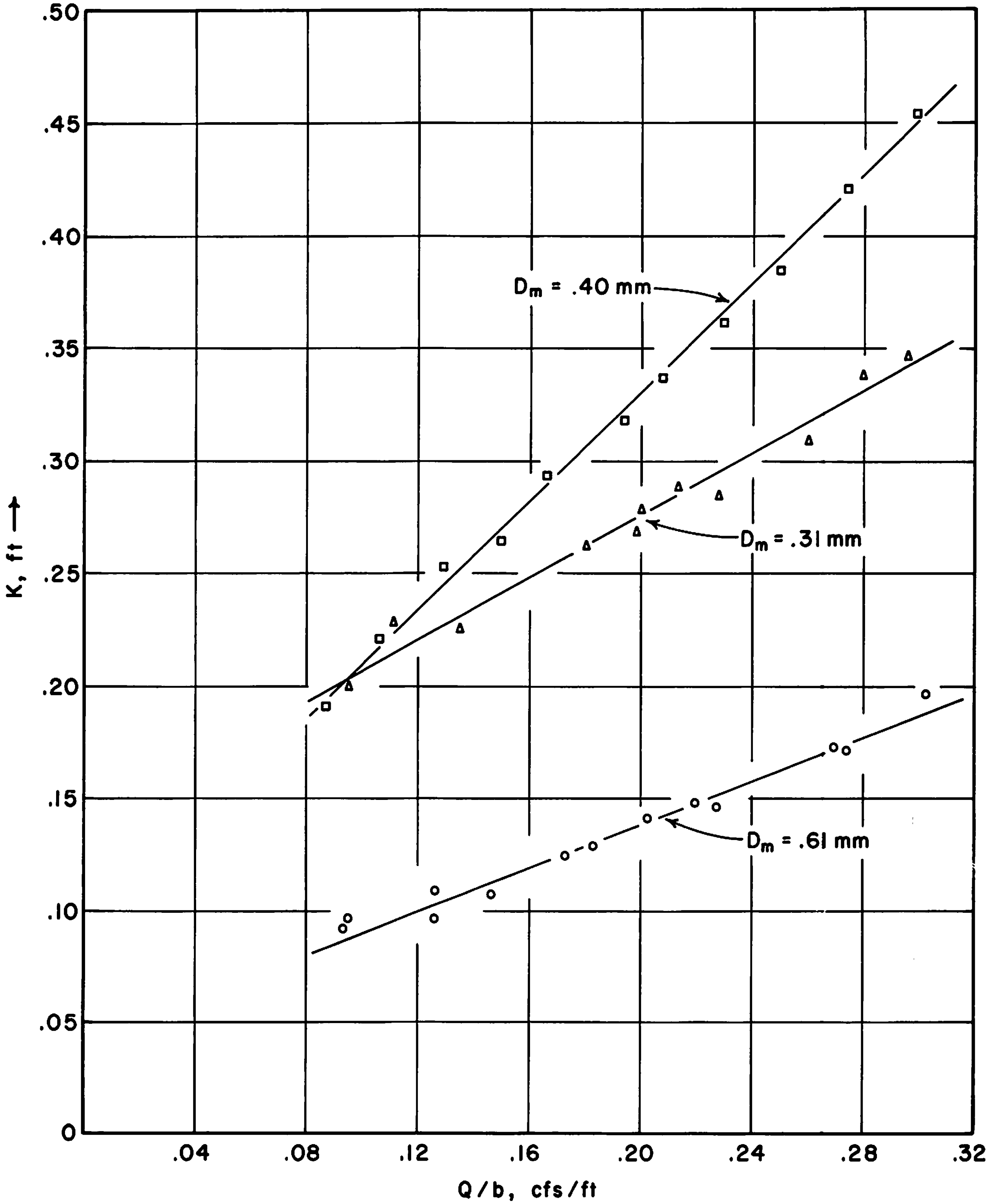

FIGURE 10.--Coefficient in bed elevation equation

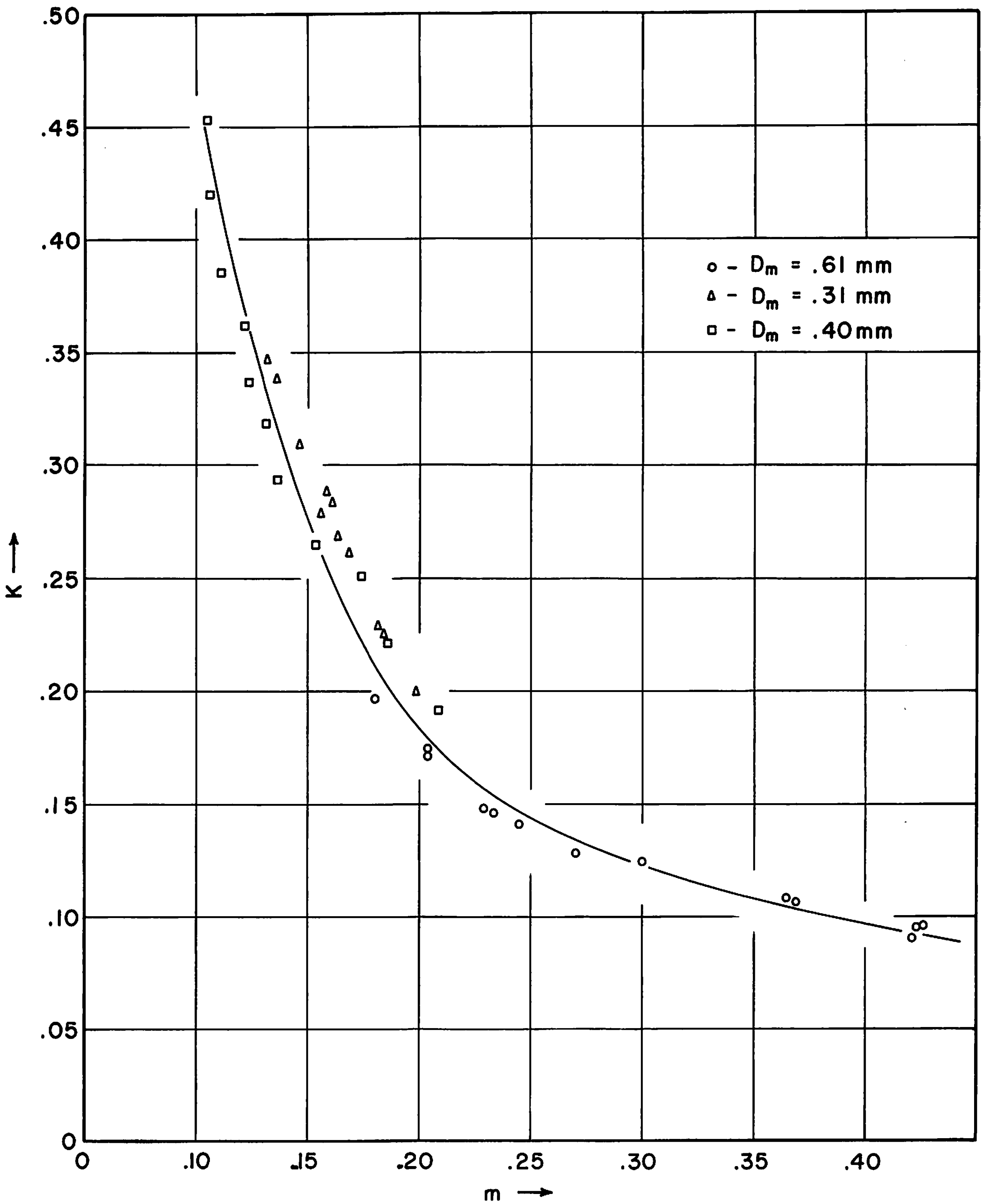

FIGURE 11.--Degradation coefficient vs exponent

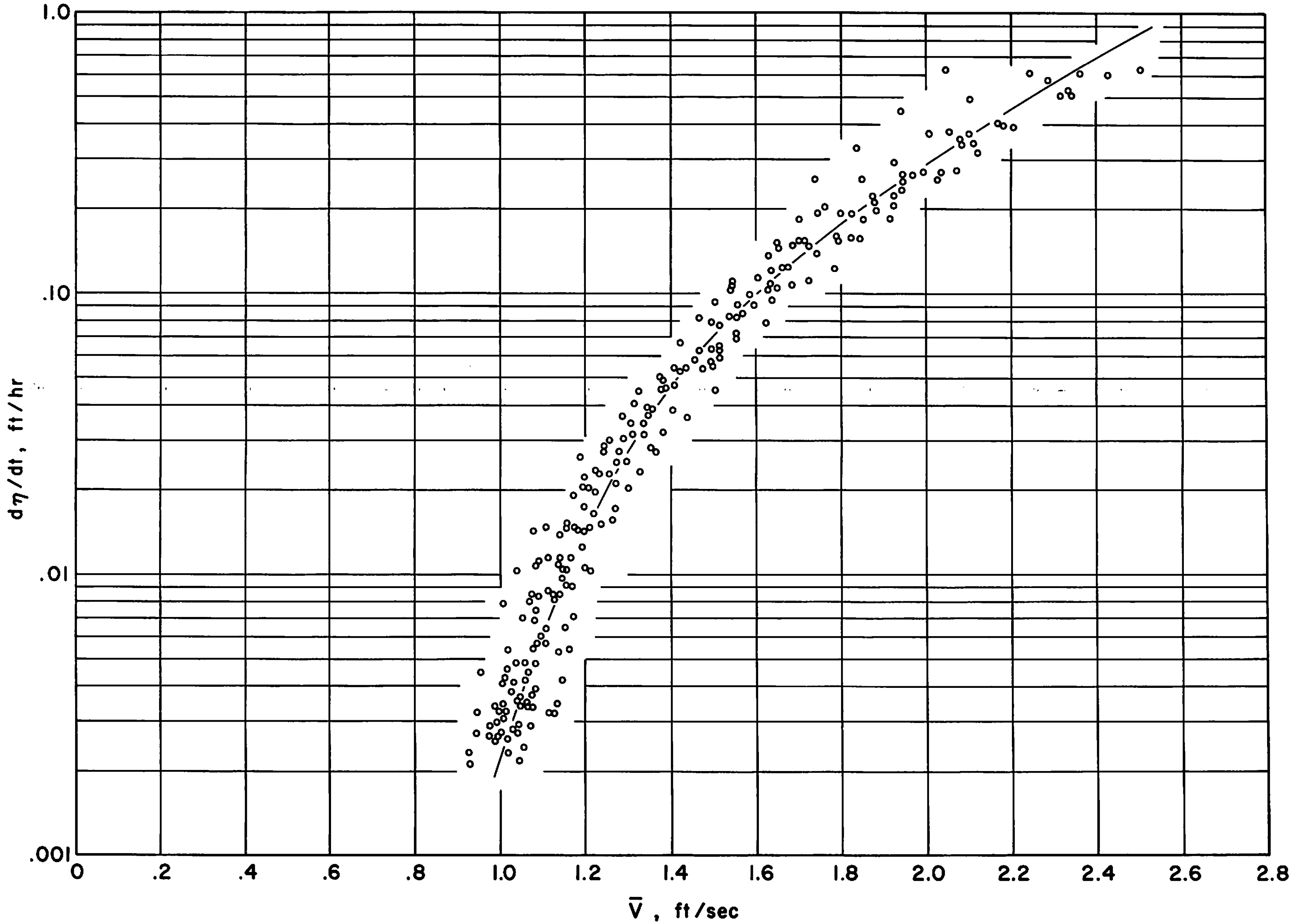

FIGURE 12.--Degradation rate vs Mean-velocity for Sand No. 1

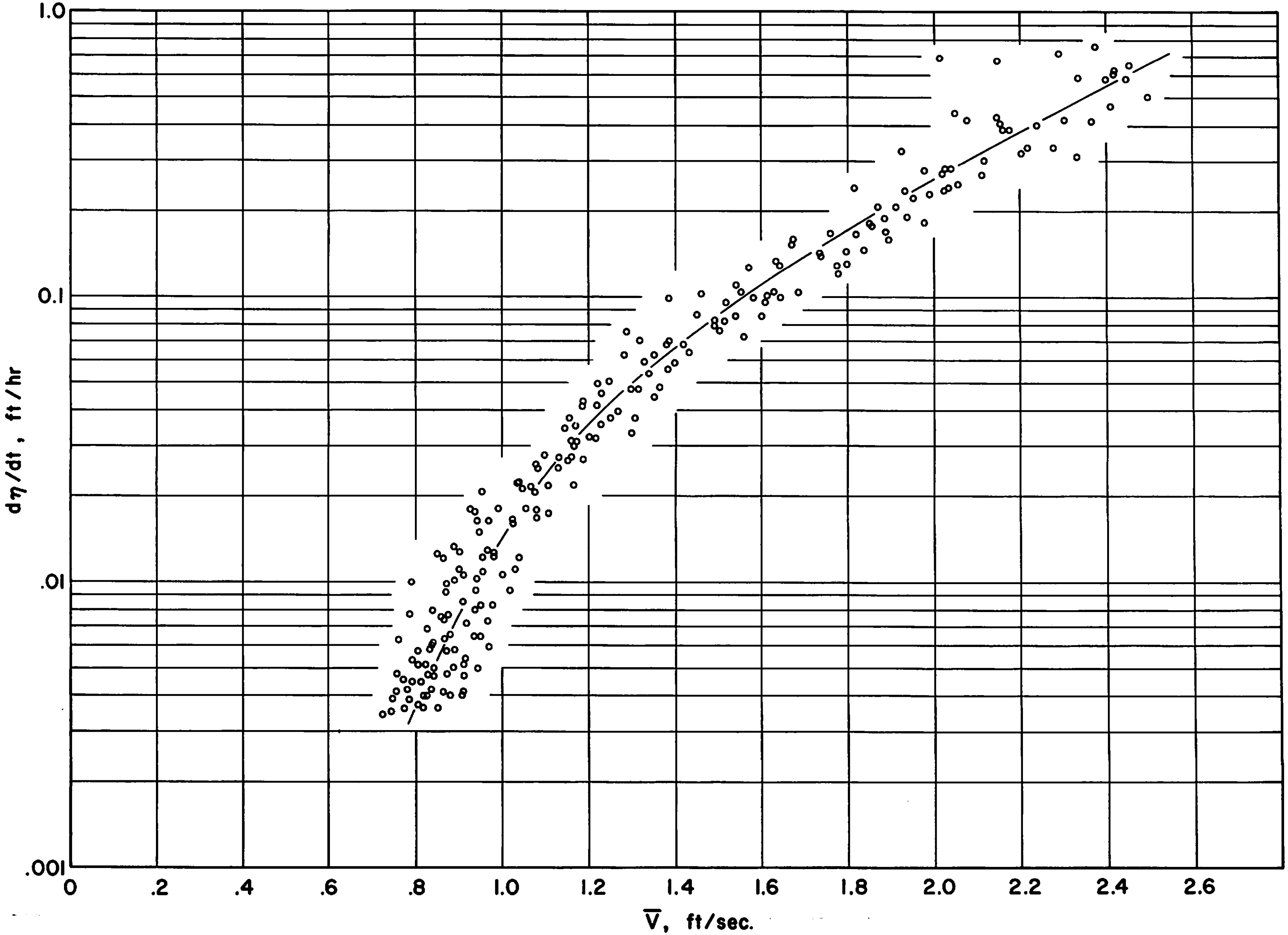

FIGURE 13.--Degradation rate vs Mean-velocity for Sand No. 2

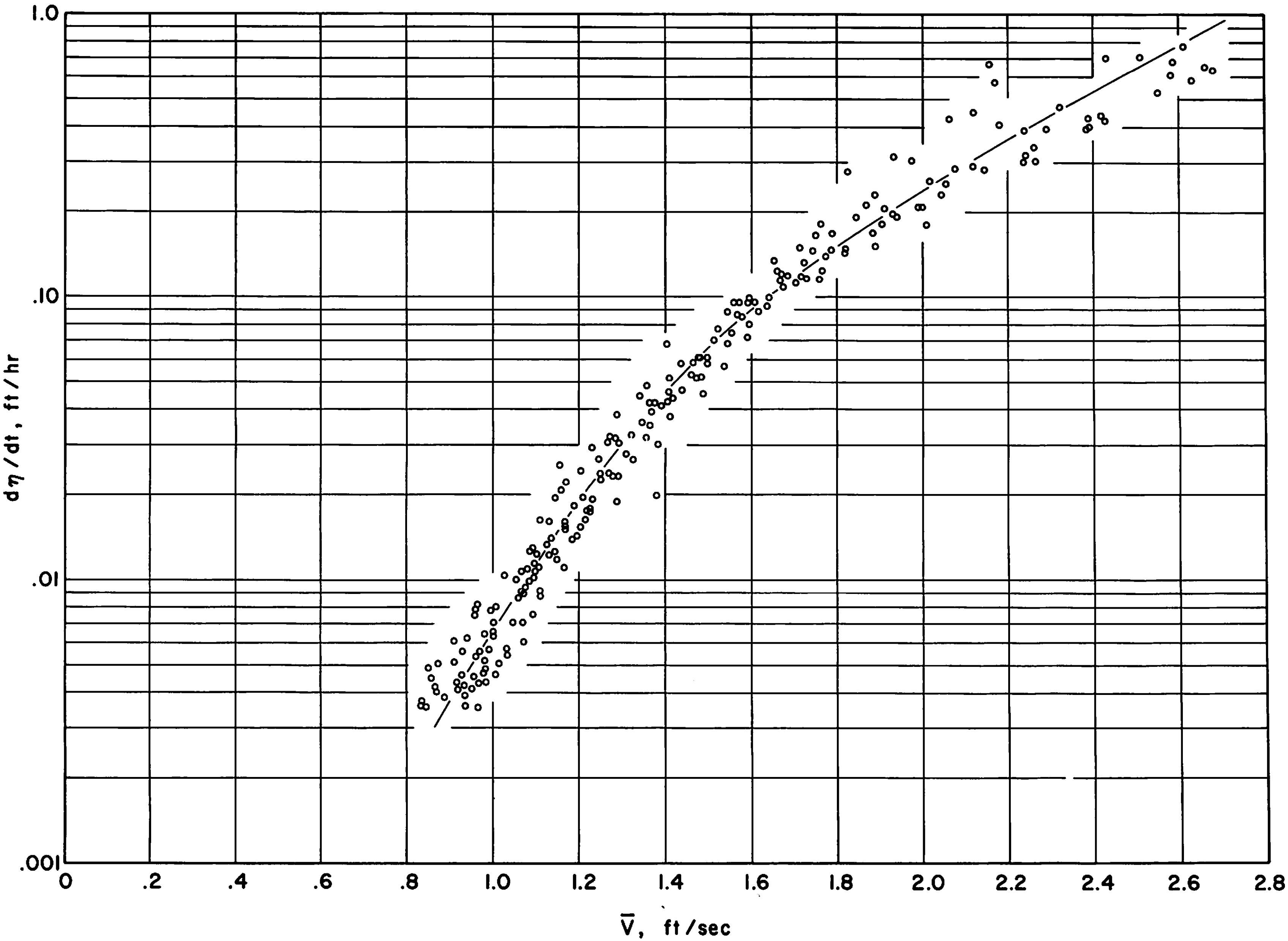

FIGURE 14.--Degradation rate vs Mean-velocity for Sand No. 3

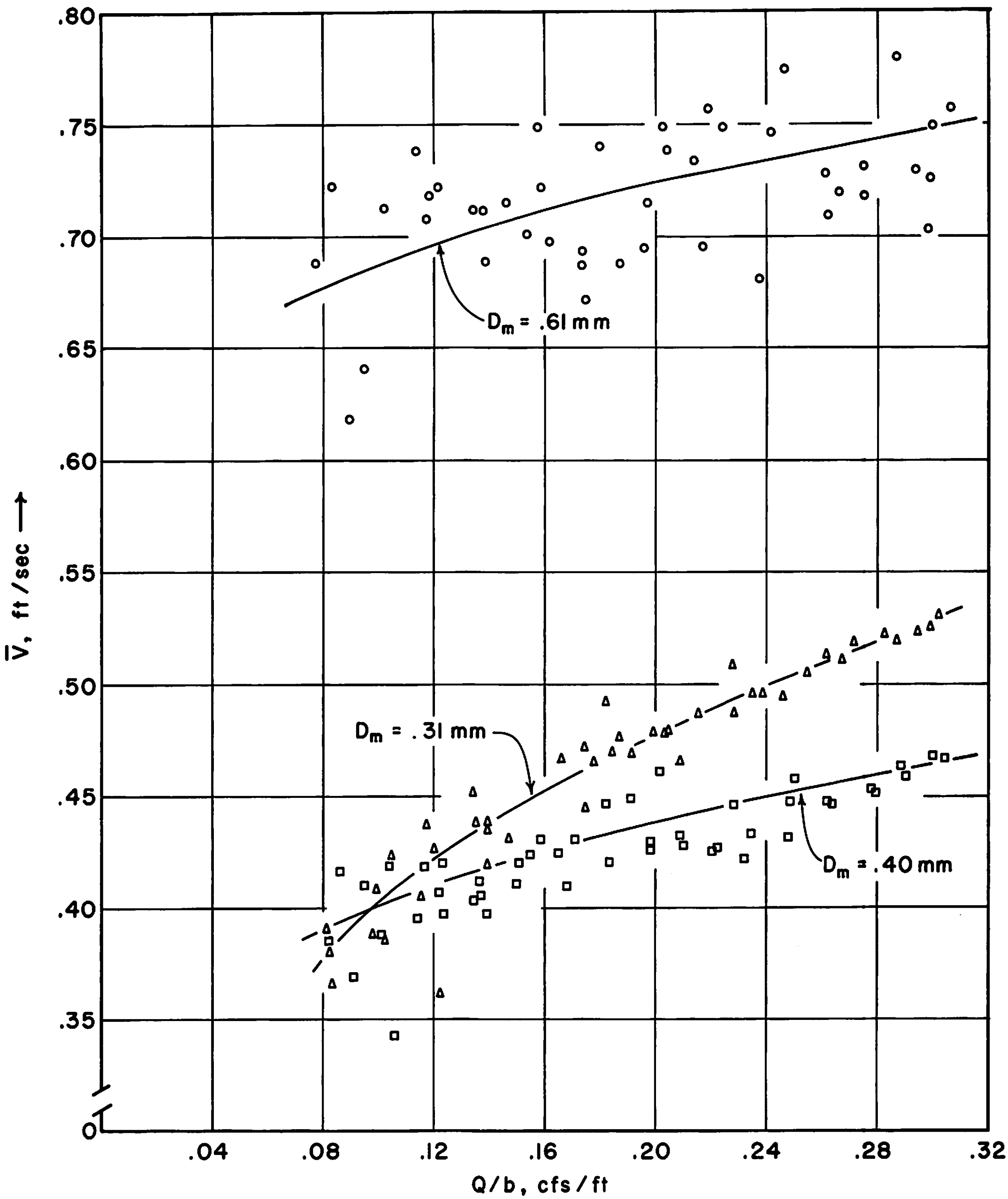

FIGURE 15.--Critical velocity for sediment motion

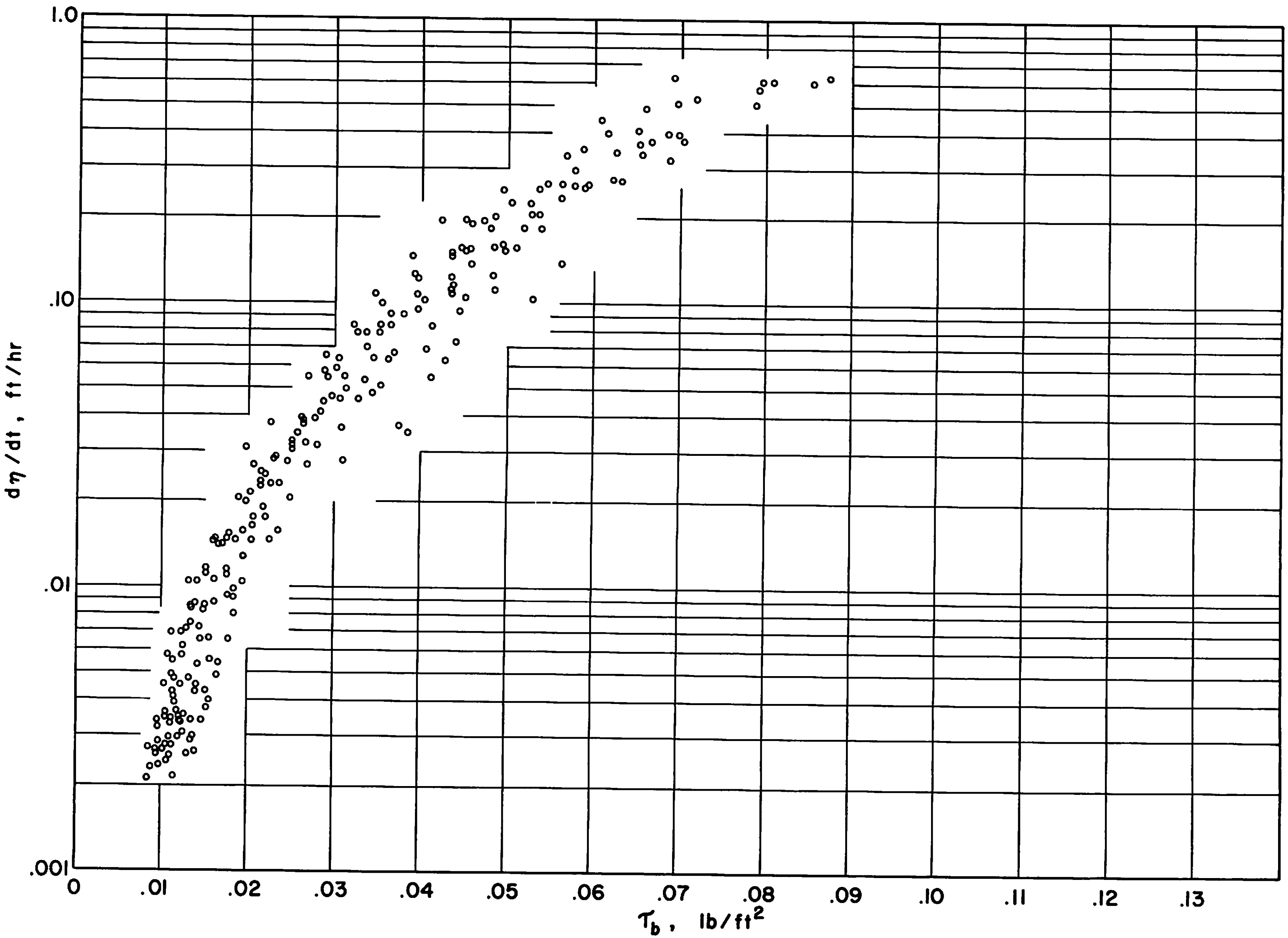

FIGURE 16.-- Degradation rate vs Tractive force for Sand No.1

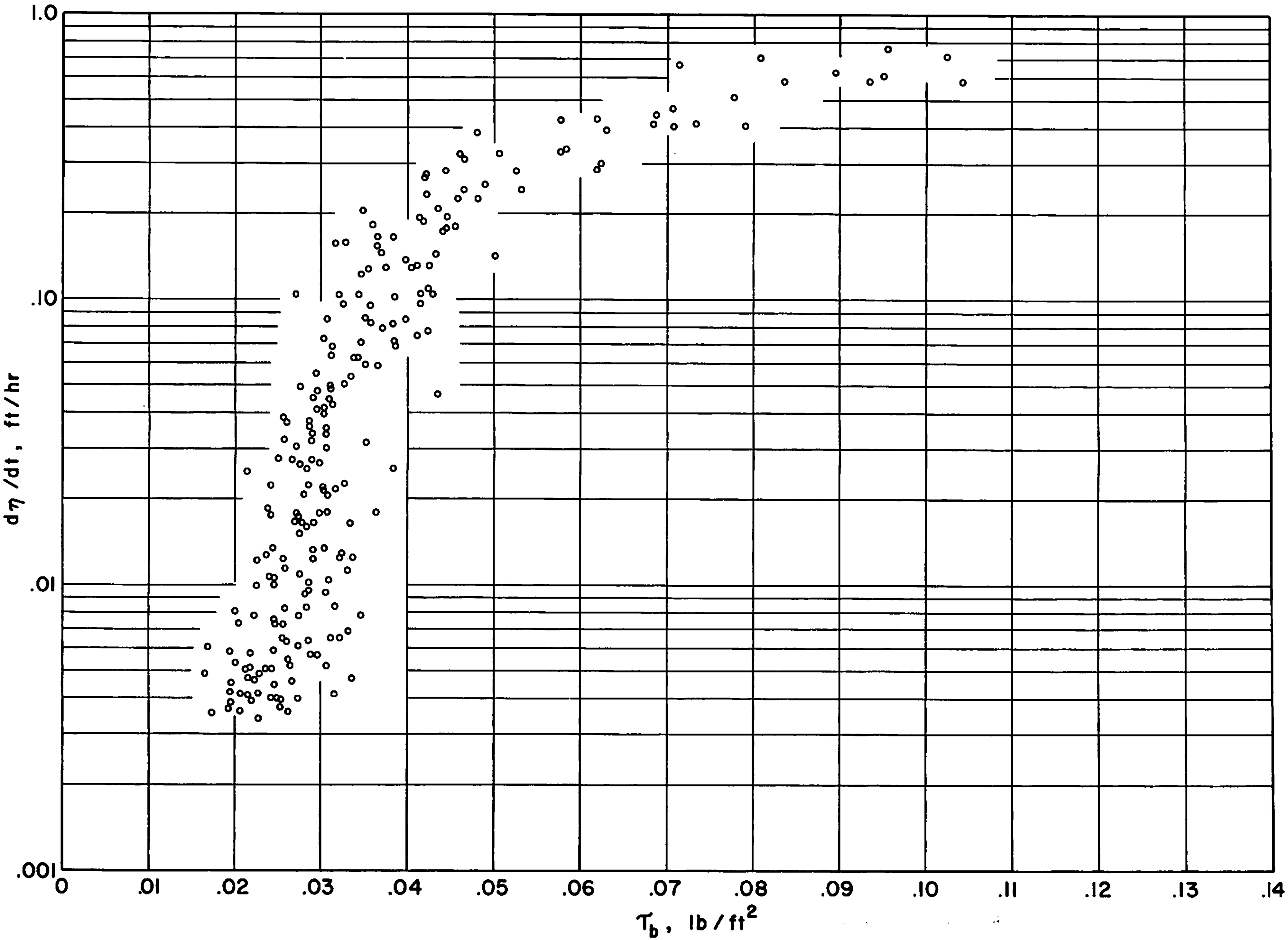

FIGURE 17.--Degradation rate vs Tractive force for Sand No. 2

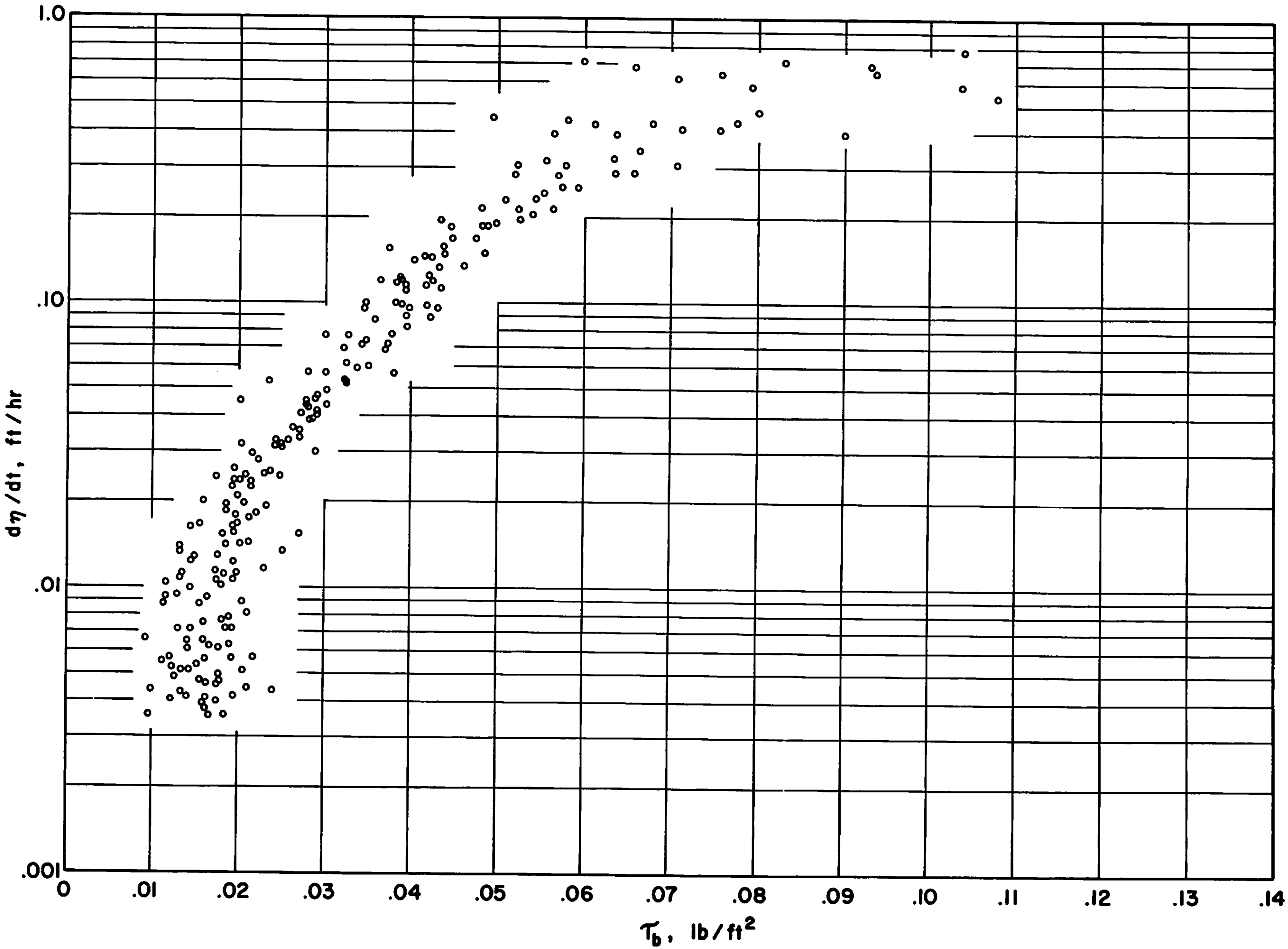

FIGURE 18.--Degradation rate vs Tractive force for Sand No. 3

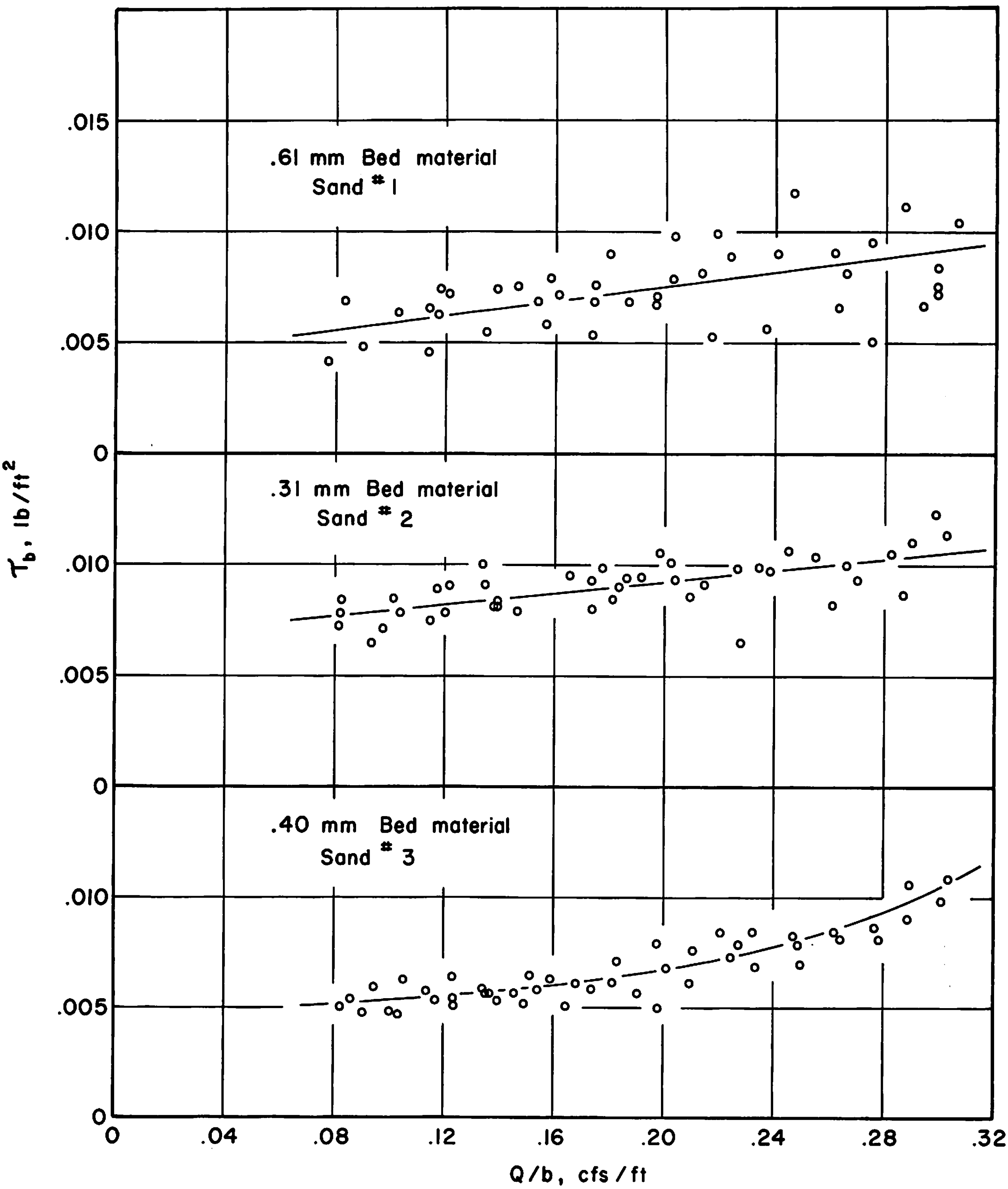

FIGURE 19.--Critical tractive force for sediment motion

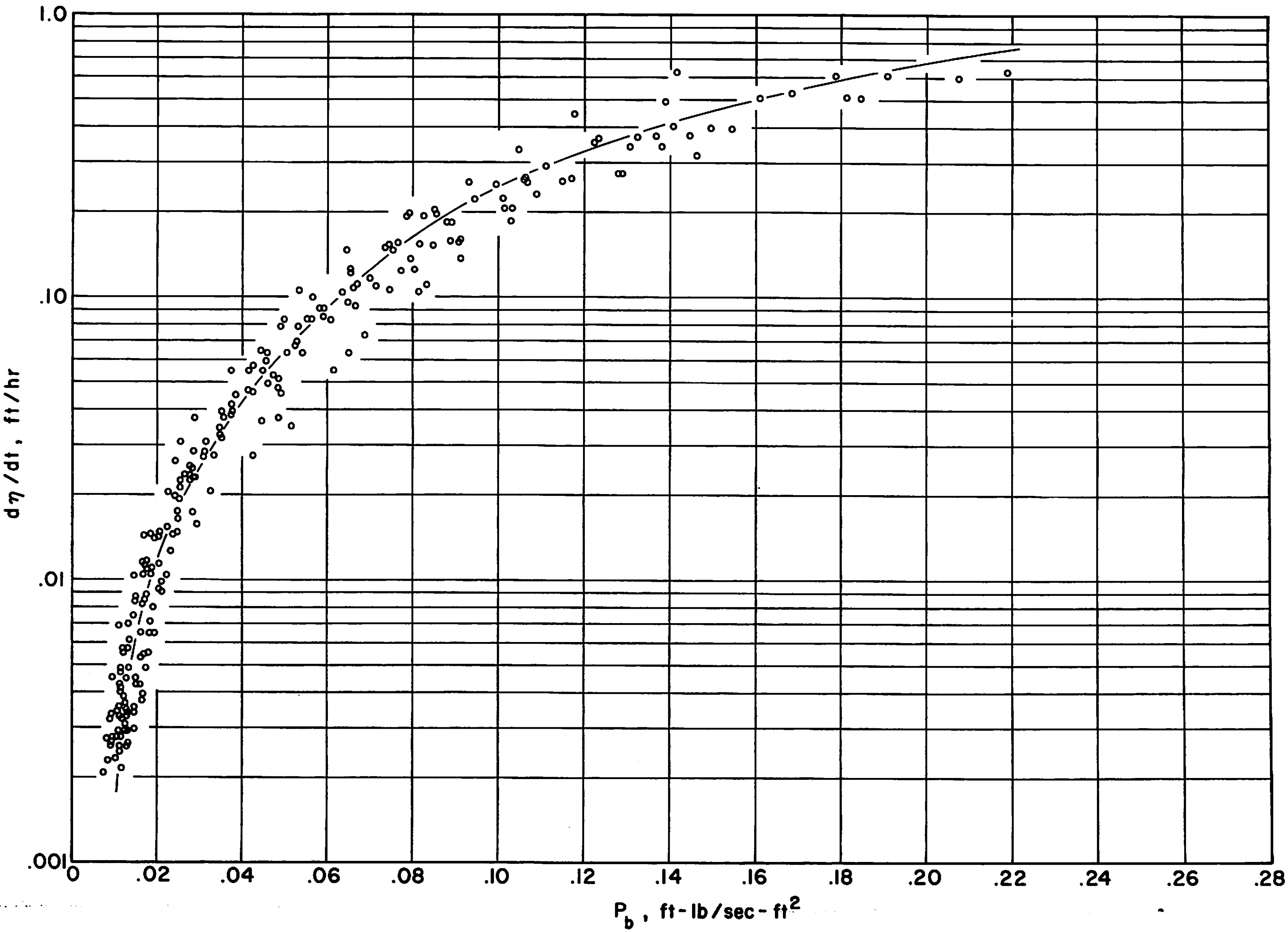

FIGURE 20.--Degradation rate vs Stream power for Sand No. 1

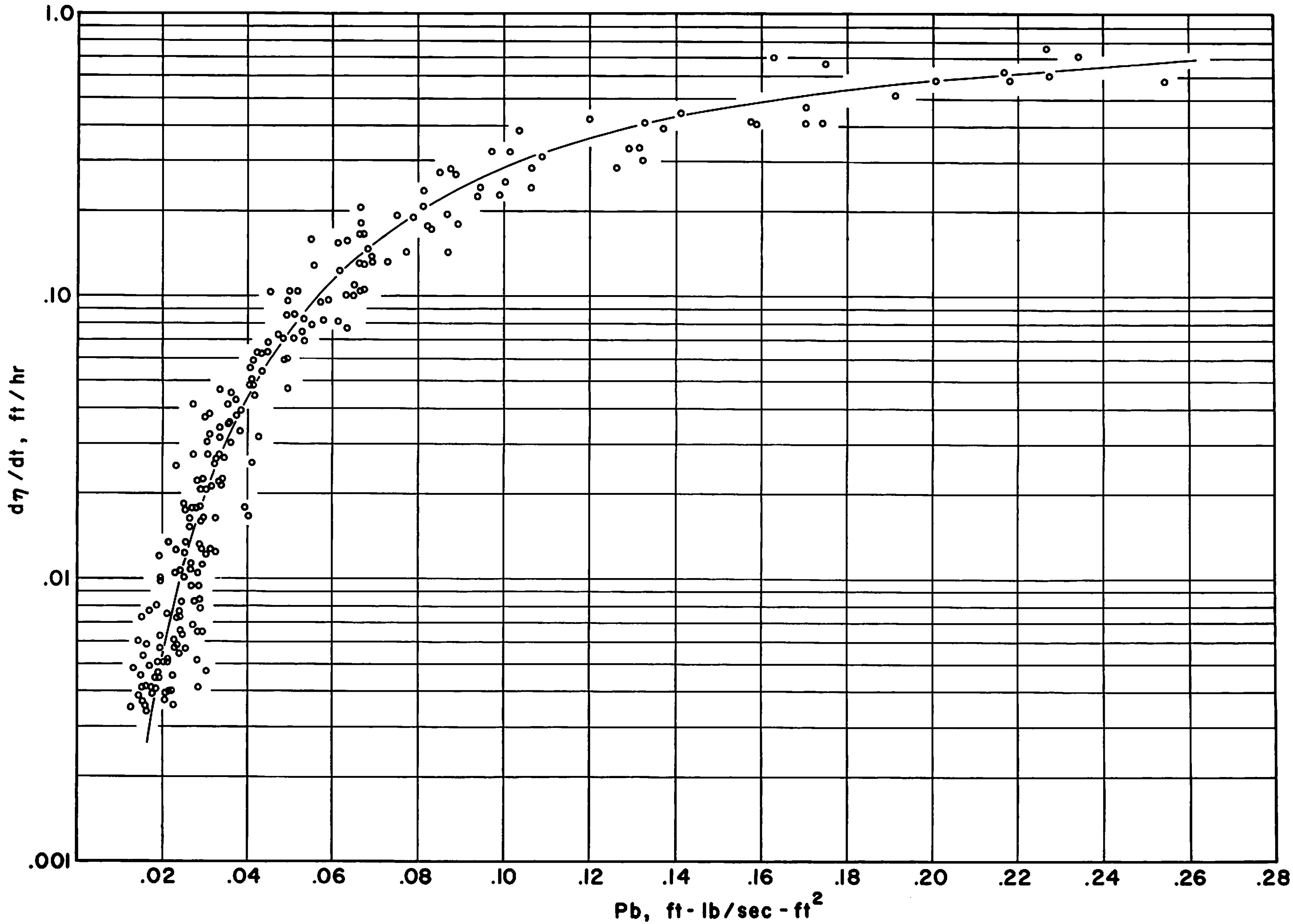

FIGURE 21.--Degradation rate vs Stream powe for Sand No. 2

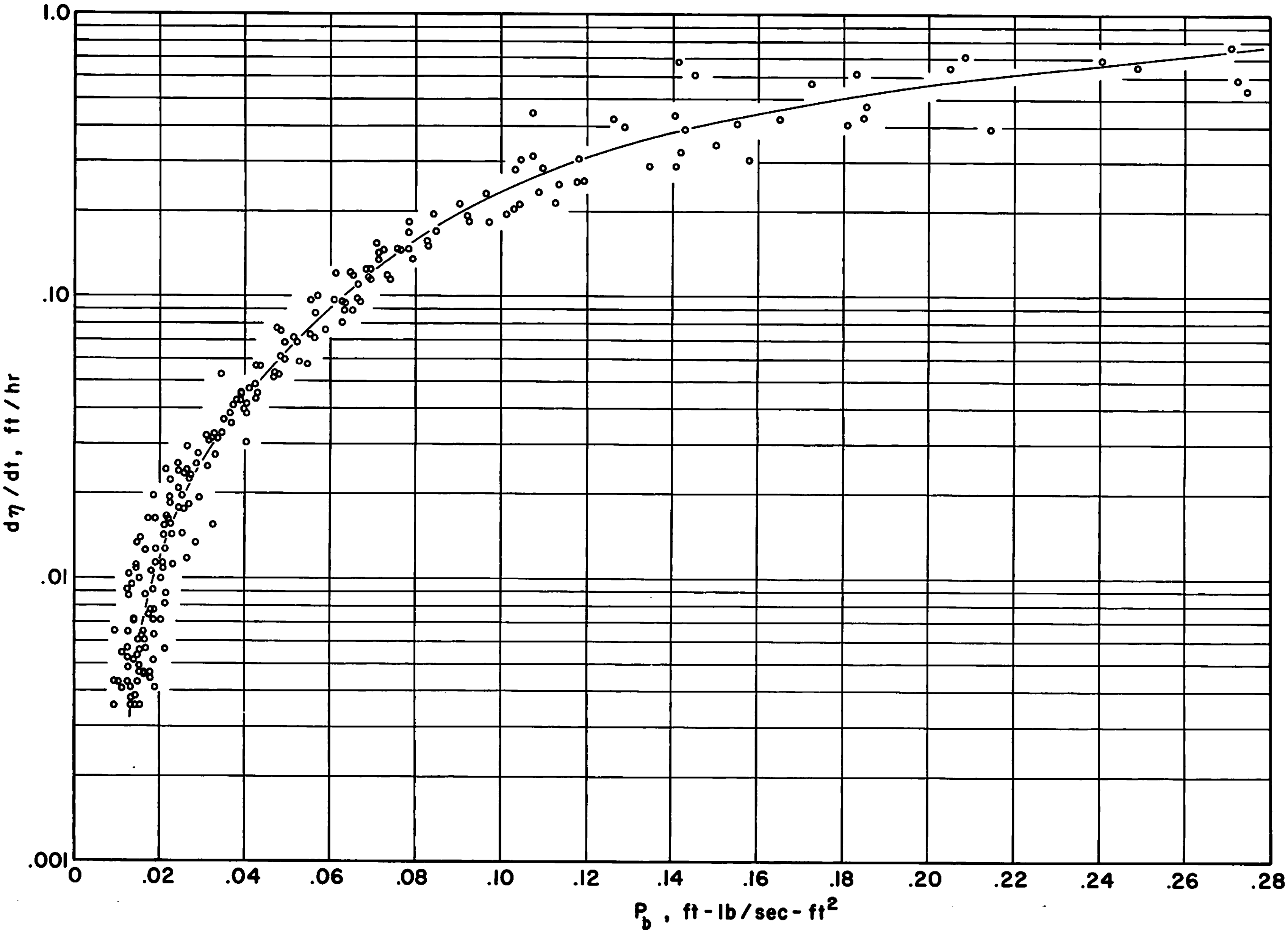

FIGURE 22.--Degradation rate vs Stream power for Sand No. 3

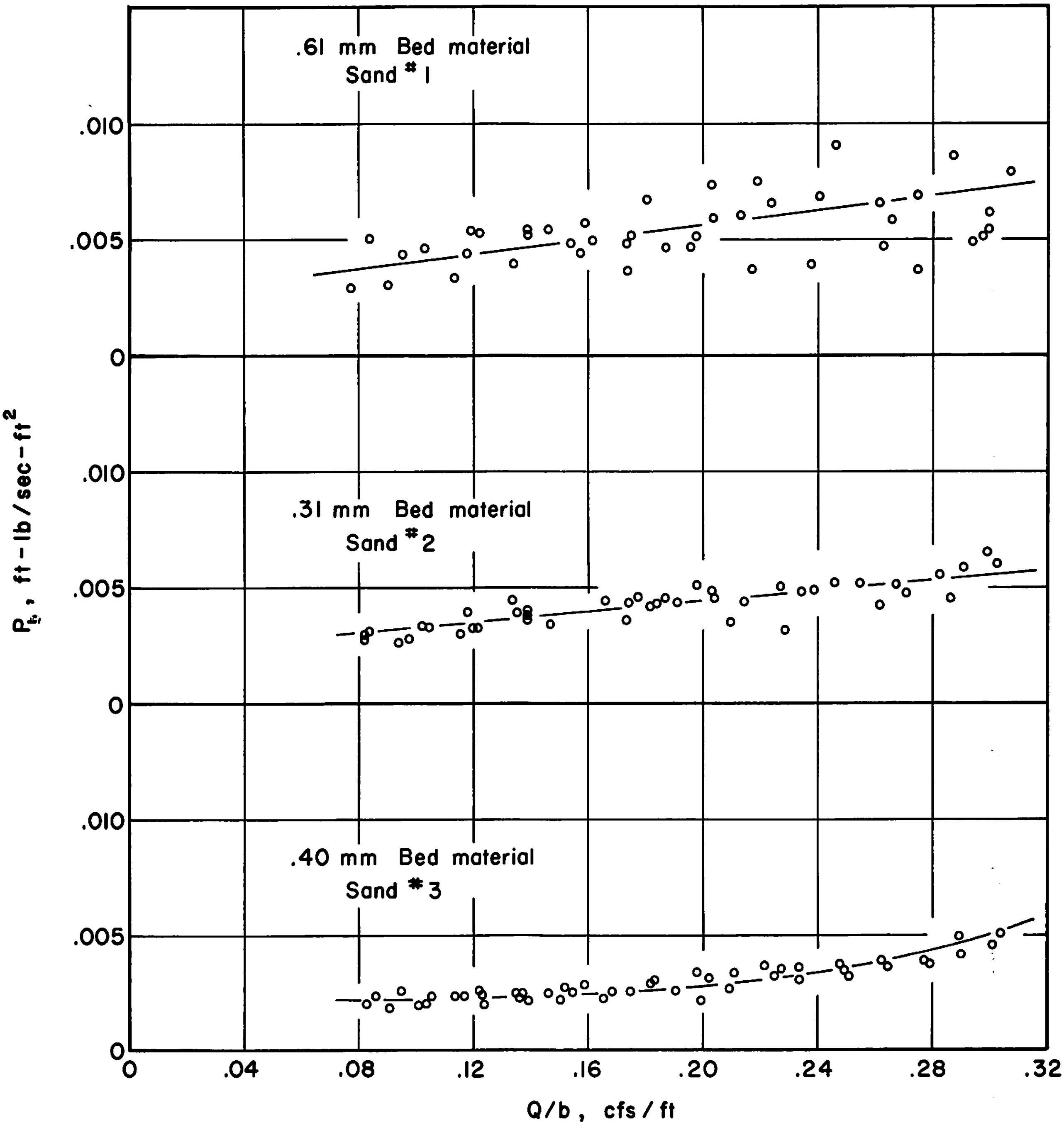

FIGURE 23.--Critical stream power for sediment motion